A Steady-State Pressure Model for Water Flow in a Hydraulically Fractured Geothermal Reservoir

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ABSTRACT

The flow of water in an Engineered Geothermal System (EGS) is an issue of considerable importance for the emerging Geothermal Industry. A steady-state mathematical model was developed for the purposes of modeling water flow through horizontal fractures of an EGS. Using classical fluid mechanics the model provided approximate values of reservoir pressure drop, injection wellhead pressure and production wellhead pressure. The model has enabled the Petroleum and Geothermal Group of the Department of Primary Industries and Resources of South Australia (PIRSA) to carry out sensitivity studies. PIRSA has used the model in its evaluation of the technical feasibility of EGS in the Cooper Basin in South Australia.

INTRODUCTION AND THEORY

The Geothermal industry in South Australia has expanded significantly in recent years. As industry regulators, the Petroleum and Geothermal Group of PIRSA has conducted independent research into the issue of fracture flow in EGS. This paper is the product of that research. A steady-state pressure model has been created to predict flow pressure drops within EGS. The model provides approximate values of required injection pressure at the injection wellhead and resulting production pressure at the production wellhead. These values in turn provide an idea as to whether EGS can operate in natural convection mode or whether it will require reinjection pumping resulting in parasitic energy losses.

The geometry of the model is shown in Figure 1. It consists of a fractured reservoir and two wells; one for injection and the other for production. Pressure drop for both the injection and production well was modeled using Equation 1 (Munson et al., 2006).

$$\Delta P_{i,p} = \rho g h_L - \rho g(z_1 - z_2) \qquad \text{(1) well flow}$$

Where $\Delta P_{i,p}$ is the pressure drop in a well, ρ is the density of water, g is gravitational acceleration, (z_1-z_2) is the distance of vertical displacement and h_L is the pressure head given by

$$h_L = f\left(\frac{D}{d_w}\right)\frac{\bar{u}^2}{2g} \qquad \text{(2) pressure head}$$

Where f is the friction factor, D is the depth of the well, d_w is the diameter of the well and $\bar{u}$ is the average flow velocity in the well. The wells were assumed completely vertical and of constant diameter. Water properties in each well were determined at constant temperature and at a pressure averaged between wellhead and wellbore. This component of the model is currently being reviewed by the author.

Flow in the fractures was modeled as flow between horizontal parallel plates (Jones et al., 1988). The fractures were assumed horizontal as over-thrust stress conditions in the Cooper Basin produce predominately horizontal fractures when granite is hydraulically stimulated (Wyborn et al., 2004). The reservoir model consists of two flow regimes: radial at the wellbores and linear in between. The model does not account for discontinuities between flow regimes. Total reservoir pressure drop was approximated by summing the pressures drops of each flow section (Slider 1983).

$$\Delta P_r = \Delta P_{lin} + 2\Delta P_{rad} \qquad \text{(3) total reservoir pressure drop}$$

Where ΔP_{lin} and ΔP_{rad} are given by equations 4 and 5 respectively.

$$\Delta P_{lin} = \frac{12\mu L' q_{pf}}{h^3 w} \qquad \text{(4) linear flow}$$

$$\Delta P_{rad} = \frac{12\mu q_{pf} \ln\left(\frac{r_e}{r_w}\right) l}{h^3 \theta} \qquad \text{(5) radial flow}$$

Equation 4 is an exact solution to the Navier-Stokes equations for steady incompressible linear flow between parallel plates (Munson et al., 2006). Equation 5 is an approximate solution to the Navier-Stokes equations for purely radial flow between parallel discs (McDonald, 2000). Water properties were determined at reservoir temperature Tr and reservoir pressure Pr. This assumption

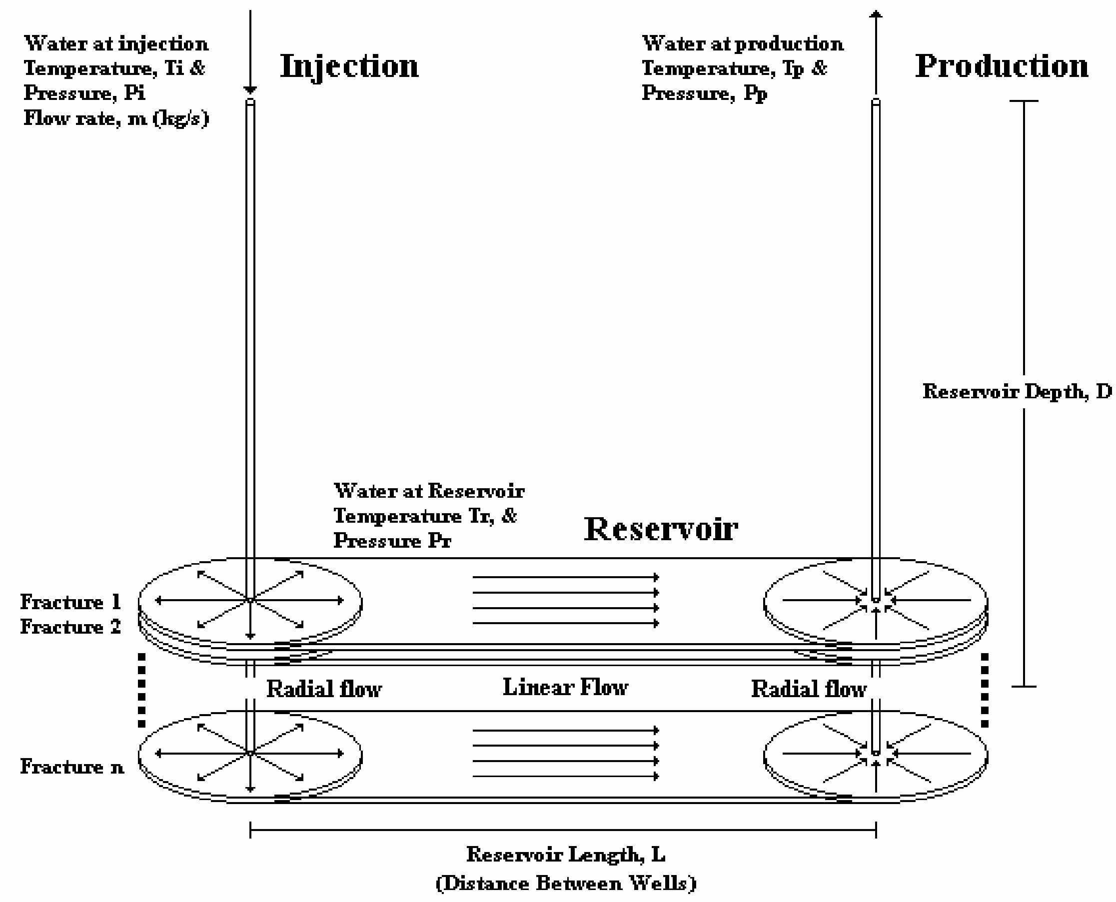

Figure 1. Geometry of EGS model.

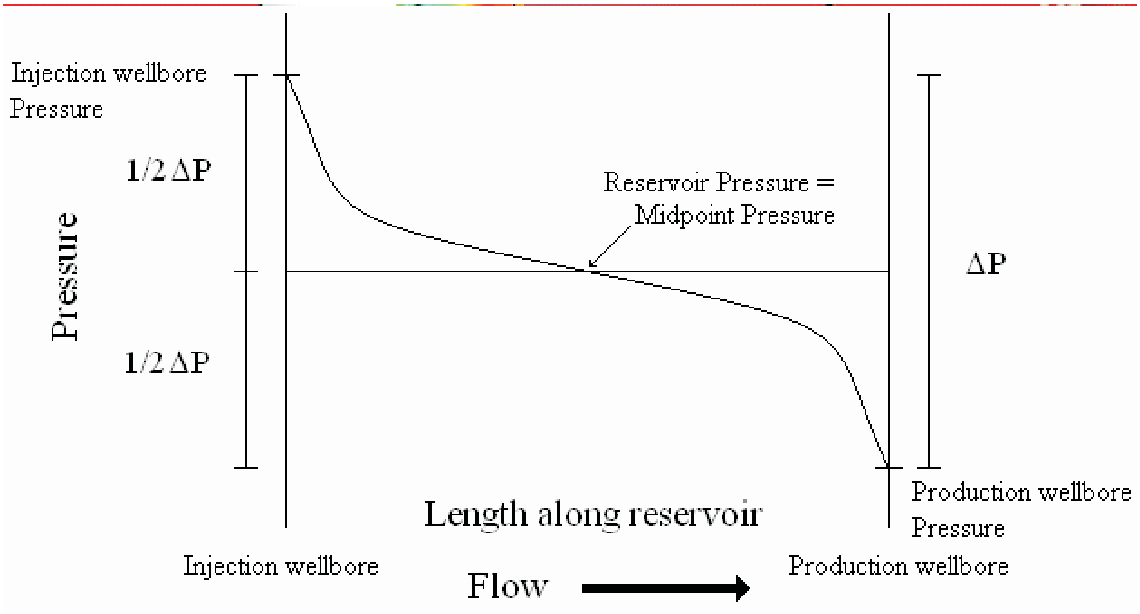

Figure 2. Reservoir pressure profile assumption expressed pictorially.

can be considered valid as heat transfer calculations have shown water to heat up to reservoir temperature not long after injection (Holman, 1997).

It was assumed that injection wellbore pressure needed to be equal to P_r plus half the reservoir pressure drop. It was also assumed that pressure at the production wellbore would be equal to P_r minus $\Delta P/2$. Figure 2 demonstrates this assumption in terms of reservoir pressure profile. Injection wellhead pressure was thus given by equation 6 and production wellhead pressure by equation 7.

$$P_i = P_r + \frac{\Delta P_r}{2} + \Delta P_i \qquad \text{(6) injection pressure}$$

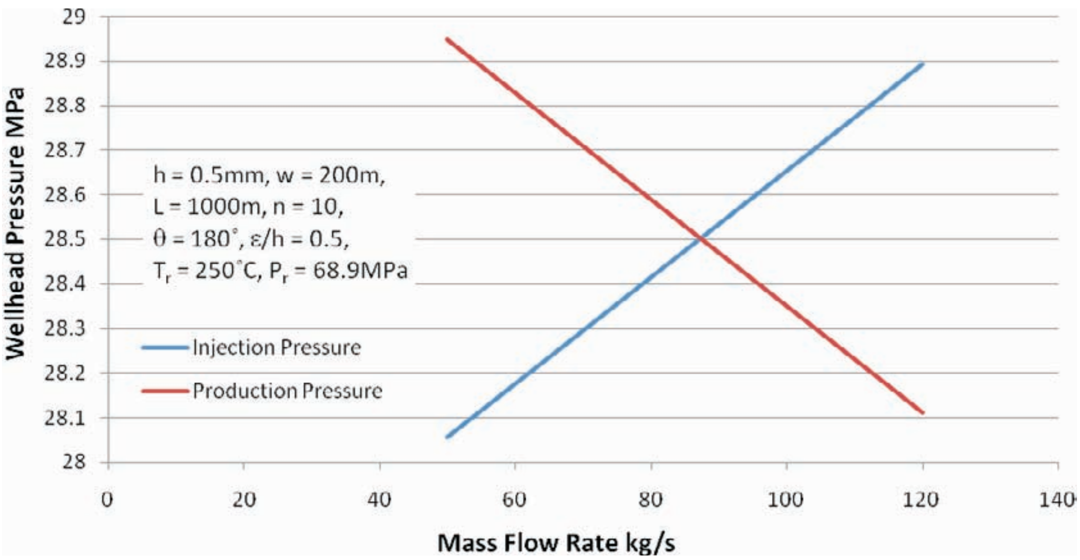

Figure 3. Wellhead injection and production pressure versus mass flow rate.

$$P_p = P_r - \frac{\Delta P_r}{2} - \Delta P_p \qquad \text{(7) production pressure}$$

The model was only able to provide solutions for laminar flow regimes in the reservoir as turbulent flow behaviour was indeterminable for radial flow. Further study is being conducted to rectify this issue. The critical Reynolds number for linear flow was

$$\mathrm{Re}_f = \frac{2h\bar{u}\rho}{\mu} \leq 2300 \qquad \text{(8) local Reynolds number for linear flow}$$

For radial flow the laminar threshold was given by the overall Reynolds number

$$\mathrm{Re}_0 = \frac{q_{pf}\rho}{h\theta\mu} \leq 1.0 \times 10^8 \qquad \text{(9) overall Reynolds number for radial flow}$$

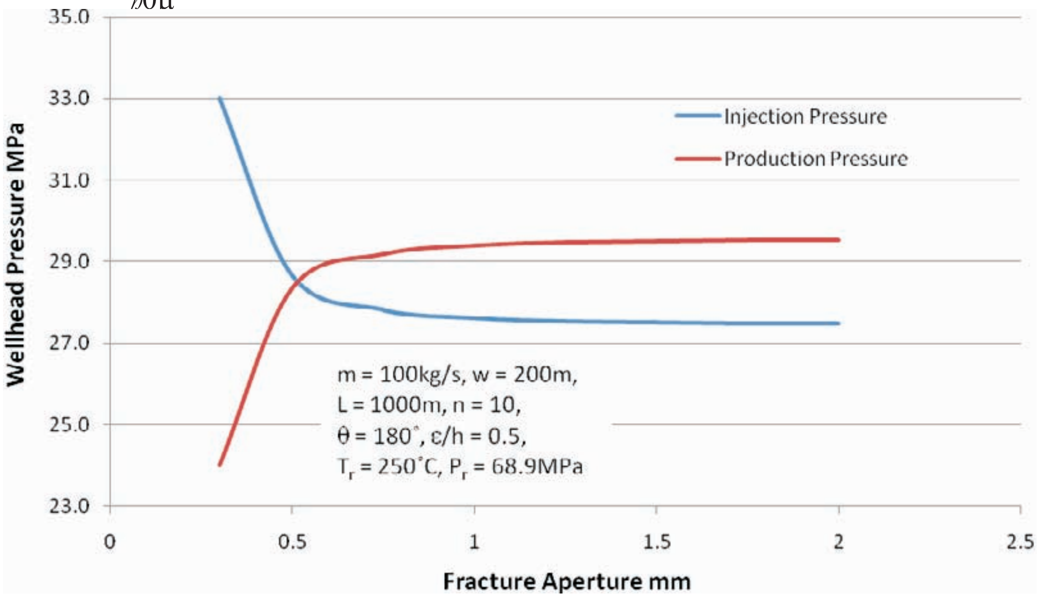

Figure 4. Wellhead injection and production pressure versus fracture aperture.

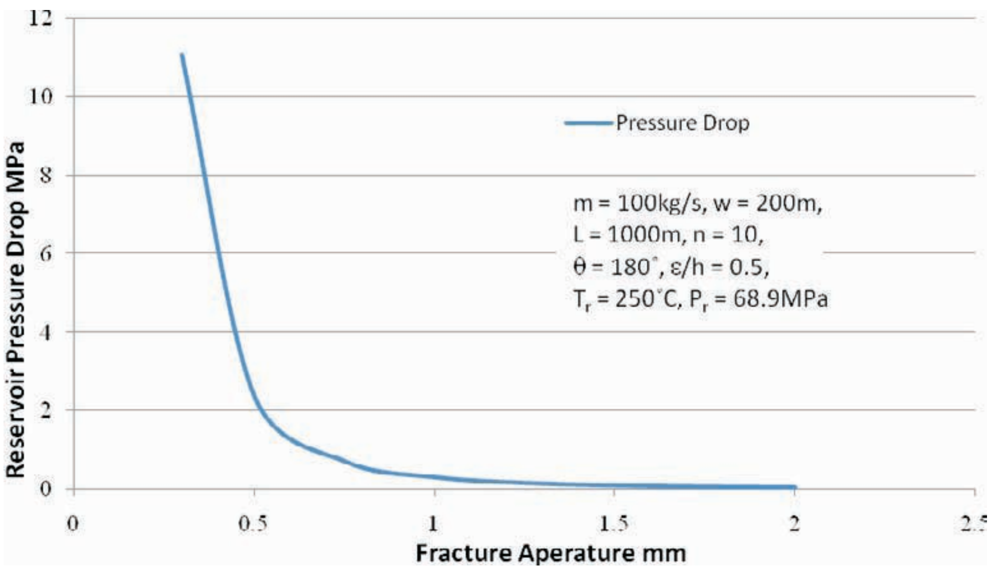

Figure 5. Reservoir pressure drop versus fracture aperture.

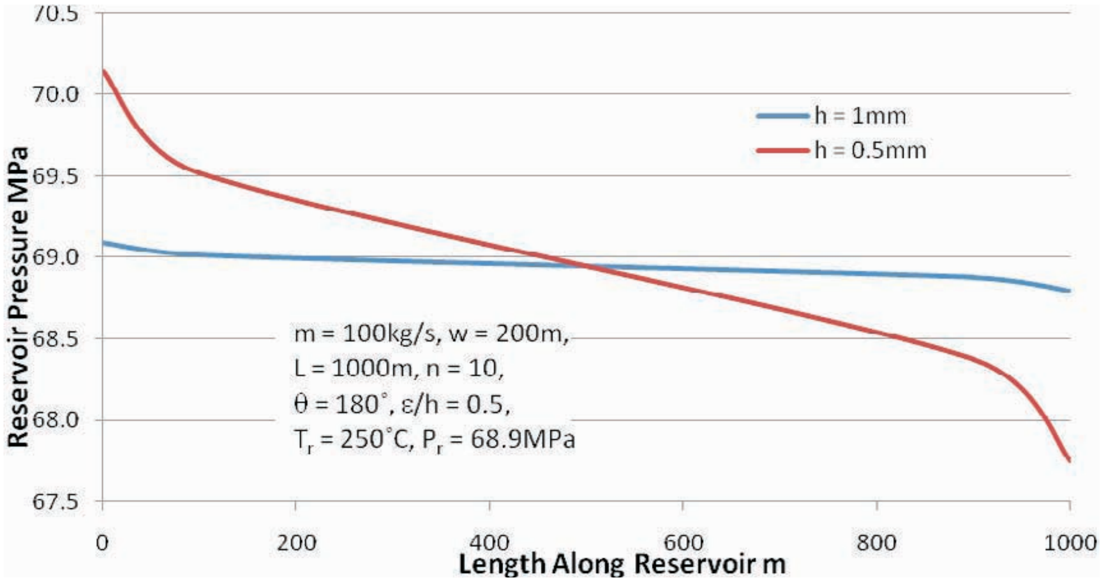

Figure 6. Reservoir pressure profile for fracture aperture 1mm and 0.5mm.

Adopted from Patel & Head (1968), the overall Reynolds number takes into account acceleration effects associated with radial flow. These acceleration effects cause flow to remain laminar despite large local Reynolds numbers (Murphy et al., 1978). The model is summarised in Table I.

Table 1. Model summary.

Modelling	Equation	Assumptions
Well flow	1 & 2	Completely vertical; and of constant diameter
Linear fracture flow	4	Completely horizontal; and uniform temperature
Radial fracture flow	5	Completely horizontal; and uniform temperature
Critical linear flow Reynolds	8	Critical value is 2300
Critical radial flow Reynolds	9	Critical overall Reynolds number is 1×10^8 due to acceleration effects
Reservoir pressure drop	3	Pressure drops are additive; and discontinuities of flow are ignored
Required injection pressure	6	Is equal to $1/2\Delta P_r + P_r$ to achieve flow into fracture
Resulting production pressure	7	Is equal to P_r - $1/2\Delta P_r$ for flow out of fracture

SENSITIVITY ANALYSIS

Using Microsoft Excel® a sensitivity analysis was conducted on the model. The effect on the model of water mass flow rate, fracture aperture and number of fractures was determined. Conservative reservoir geometries at a depth of 4,500 m were used for the analysis. Additionally a reservoir temperature of 250 °C was assumed whilst a reservoir pressure of 68.9 MPa (10,000 psi) was used to simulate the overpressure conditions of the Cooper Basin. However, the model can be used to simulate reservoirs without the existence of overpressure.

RESULTS AND DISCUSSION

Figure 3 shows the effect of mass flow rate on pressure at the injection and production wellheads. Due to the mathematics of the model the relationship between mass flow rate and pressure is

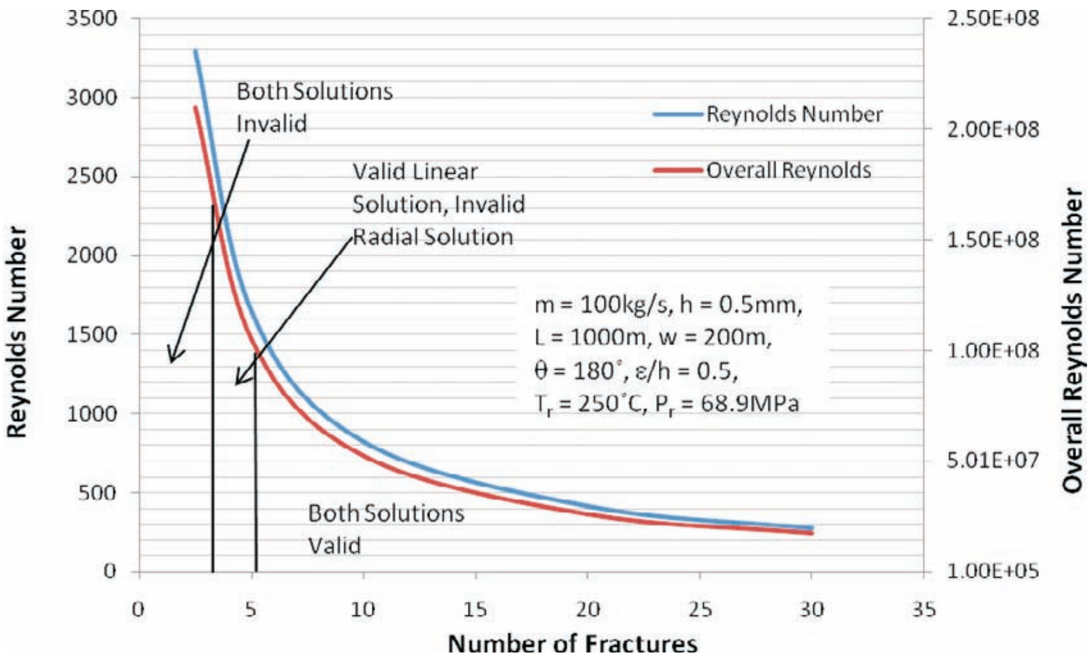

Figure 7. Reynolds number and overall Reynolds number versus number of fractures.

linear. It can be seen on the chart that up until a certain point production pressure exceeds injection pressure. This result can be reasoned by the model assuming that water in the production well is at a higher temperature than the injection well. The less dense water being produced is therefore flowing against less hydrostatic head than the amount of head that is gained with higher density water flowing in the injection well. As a consequence it may be possible to operate EGS as a naturally convective system, that is, parasitic energy losses would be minimised.

Figure 4 shows the effect of fracture aperture has on wellhead pressures. It can be seen that for certain fracture apertures production pressure can exceed injection pressure. Again this is due to less dense water flowing in the injection well than in the production well coupled with increasing fracture aperture which results in decreasing reservoir pressure drop. This second factor is demonstrated in Figure 5 which shows reservoir pressure drop falling significantly with increasing fracture aperture. Figure 5 therefore also emphasises the importance of obtaining a good fracture network.

The model was able to determine flow pressure at any point along the reservoir. Figure 6 shows the reservoir pressure profile for fracture aperture equal to 1 mm and 0.5 mm along the length of the reservoir. It can be seen that the greatest pressure drops occur around the wellbore where the slope of the profile is greatest. This is due to increasing velocity as the water approaches the wellbore.

The model obtained solutions for most reservoir geometries. Turbulent flow would only be expected in cases of geometries which would render a reservoir uncommercial; that is pressure drops would be too high. Figure 7 shows the applicability of the model with respect to number of fractures in the network.

CONCLUSIONS AND RECOMMENDATIONS

The model showed that, for given reservoir geometries, it is possible for wellhead production pressure to exceed wellhead injection pressure. This was reasoned to be the result of less dense and less viscous water flowing in the production well than in the injection well. This means that it may be possible to flow the injection well without the assistance of an injection pump thereby avoiding the parasitic energy losses of running the pump. This preferred operational mode may be called naturally convective. The reservoir pressure profile plots showed that the larger pressure drops within the reservoir occur at the wellbores where the flow regime is radial. The sensitivity analysis on reservoir pressure drop demonstrated that fracture aperture is the most important element of a geothermal reservoir with respect to flow. This emphasised the requirement of a well fractured reservoir to operate EGS effectively. The model was found to be applicable to many reservoir geometries. It was inapplicable for reservoir geometries that would not be considered commercially viable.

The model is simplistic but it is a good basis for further sophistication and refinement. Further study will be conducted commencing July 2008 to investigate non-isotropic well flow and to incorporate a heat exchanger at the surface. In addition the model will be compared to pressure data in literature and altered to model flow for a five-spot well arrangement.

NOMENCLATURE

Symbol	**Meaning**	**Units**
D	Well depth	m
ΔP	Pressure drop	psia
d_w	Well diameter	m
ε	Roughness	m
f	Friction factor	-
g	Gravitational acceleration	m/s^2
h	Fracture aperture	m
h_L	Pressure head	m^2/s^2
L	Distance between wells	m
L'	Length of linear flow section	m
m	Total mass flow rate	kg/s
μ	Fluid dynamic viscosity	Pa.s
n	Number of fractures	-
v	Kinematic viscosity	m^2/s
P	Pressure	Psia
q	Volumetric flow rate	m^3/s
θ	Angle of radial flow	°
R_e	Reynolds number	-
ρ	Fluid density	kg/m^3
T	Temperature	°C
$\bar{u}$	Average velocity	m/s
w	Fracture width	m
z	Vertical Displacement	m

Subscripts

avg	Average
b	Bulk
e	External
f	Fracture
i	Injection
lin	Linear flow
o	Overall
p	Production

Symbol	Meaning	Units
pf	Per fracture	
rad	Radial flow	
r	Reservoir	
w	Well	

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