MATHEMATICAL MODELLING OF HEAT AND MASS TRANSPORT IN DEEP GEOTHERMAL RESERVOIRS

Graham J. Weir

Applied Mathematics
The New Zealand Institute for Industrial Research and Development
PO Box 31-310, Lower Hutt
New Zealand

ABSTRACT

Current mathematical modelling of deep geothermal processes in New Zealand include generic supercritical chemical research, single field pluton/hot plate modelling, and conceptual multi-field modelling. This paper concentrates on a conceptual model of Taupo Volcanic Zone, (TVZ), New Zealand, emphasising the implications of the findings of recent GPS and micro-seismic studies in TVZ on models of processes transporting mass, heat and chemicals. We argue that in addition to the well established process of groundwater convection extracting heat and chemicals by interacting with magmatic-like intrusives under TVZ, that two other processes may be important. Firstly, the existence of a ductile layer between about 7 to 15 km depth will produce a region of "enhanced conduction" in which very high conductive fluxes of energy arise from a temperature distribution which varies exponentially with depth. Secondly, water may transport up through the ductile layer, episodically, as a result of extensional processes in the ductile region. Heat transfer in the TVZ of about 4200 MW may be made up from about 1300 MW from the cooling of intrusives in the brittle region in the upper 7 km; of about an additional 2300 MW of conducted heat entering the brittle region from the ductile region; and about an additional 600 MW of episodic water transport through the ductile region.

Keywords heat flow, deep geothermal fields, extension, conduction

TABLE 1
Approximate TVZ parameters

Symbol	Meaning	Assumed value
v	average extensional speed	10 mm/year
z	vertical coordinate	
z_m	"linear" ductile thickness	2 km
C	Magma specific heat	1.3×10^3 J/kg/C
D	Thermal diffusivity	5.9×10^{-7} m^2/s
H	Brittle thickness	7 or 15 km
J_K	Equation (4)	0.47 W/m^2
K	Rock conductivity	2 W/m/C
L	Magma crystallization energy	3.5×10^5 J/kg
S	TVZ length	160 km
T_f	Brittle/fracture temperature	400°C
T_m	Melting temperature	1200°C
W	TVZ width	30 km
Y	Ductile layer thickness	8 km
δ	Scaled ductile layer thickness	7.3 km
ρ	Rock density	2600 kg/m^3
ΔT	Temperature difference (1)	800°C

1. INTRODUCTION

The aim of this paper is to summarise some of the current research in New Zealand on the mathematical modelling of deep geothermal reservoirs. We begin by briefly outlining generic chemical modelling research; and modelling of single plutonic intrusions. We then discuss the TVZ region of New Zealand, and develop a conceptual multi-field model of the TVZ.

2. GENERIC CHEMICAL RESEARCH

Deep geothermal reservoirs may extend to depths of perhaps 8 km, or greater, and involve pressures of the order of 1000 bar, and temperatures up to 800°C. Two methods are being used to extend chemical equilibrium software to these extreme supercritical regions. Chemical equilibria data is missing for most of this region of phase space. However, once temperatures are above about 300°C, then if ionisation is small, chemical reactions may be relatively unimportant, and the key factors in modelling may be determination of phase equilibria and the corresponding physical phase properties such as density, viscosity and enthalpy.

2.1 Empirical Phase Equilibrium Relationships

McKibbin and McNabb (1993) have constructed empirical relationships describing the phase equilibria for H_2O - NaCl, and McKibbin and Palliser (personal communication) are well advanced in determining the corresponding H_2O - NaCl - CO_2 relationships, for temperatures to 800°C and pressures of up to 1000 bar. Their research approximates the phase diagram for these systems, and approximates the corresponding physical properties of the phases.

2.2 Equation of State Thermodynamics

White and Nicholson (personal communication) have just completed an Equation of State for an H_2O - NaCl system, up to 5000 bars and 1200°C, based on work of Anderko and Pitzer (1993). Densities and enthalpies are constructed from the EOS, and viscosities are calculated independently using correlations of Phillips et al. (1981).

2.3 Two Phase/Supercritical Chemical Transport

The chemistry of surface features can indicate deep flow processes, and by modelling such features, information may be determined on deep transport. For example, White (personal communication) has extended previous work (White and Kissling, 96) to include sulphur and CO_2 chemistry to model acid sulphate and steam-heated bicarbonate pools. The relative location of these two types of pools depend on the degree of cross-flow in the reservoir.

3. PLUTON INTRUSION MODELLING

Numerical models have been constructed by Kissling (1996) of the transport of mass and energy from depths of 8 km. A given magmatic water flux and energy flux are specified at a depth of 8 km, equal to the measured surface outflow values. The temperature of the base, and of the rising plume, are calculated. Groundwater is entrained by the rising geothermal plume. The width of the intruded pluton was varied, together with the permeability structure, until the plume temperature and the magmatic to groundwater ratios are approximately those measured at a depth of about 3 km. Calculations to date have imposed cylindrical symmetry, and suggest that quite low permeabilities (about 1 mD) exist below the TVZ.

4. CONCEPTUAL MODEL FOR TVZ

The Taupo Volcanic Zone (TVZ) is in the North Island of New Zealand, extending in a north-south direction (Cole, 1990), and opening in the north to be about 60 km in east-west extent (Grindley, 1965). TVZ consists of a northern segment containing the andesitic volcano White Island; a central region between Edgecumbe and Taupo containing mostly rhyolitic volcanism; and a southern andesitic region containing Mt Tongariro, Ngauruhoe and Ruapehu. The geological, geochemical and geophysical properties of TVZ have recently been summarised in a Special Issue (Simmons and Weavers, 1995) on TVZ.

Hochstein (1995) has emphasised that the long term eruption rate of rhyolites from TVZ is the highest on earth for a volcanic arc setting, and also that the total crustal heat transfer is arguably the highest on earth for an arc setting. A widely accepted model of crustal heat transfer in TVZ (Stern, 1987) is that the region of extension within TVZ is occupied by high temperature magma, which is eventually cooled by groundwater. The heat source from these intrusives is then equal to

$$S\,H\,v\rho\,(L+C\Delta\,T) \qquad (1)$$

where S is the slip distance (160km), H depth (15 km), v extension rate (12 - 20 mm/year), ρ rock density (2600 kg/m^3), L latent heat (3.5 10^5 J/kg), C heat capacity (1300 J/kg/°C), and ΔT is the temperature difference (800°C) between molten rock and rock cooled by convecting groundwater. The heat production rate is therefore between 3400 and 5800 MW, which straddles the measured total output of 4200 MW over an area of 5000 km^2.

The "intrusive" model in (1) has the great advantage that it predicts that the chemical output from TVZ will reflect the composition of the emplaced magmas. Extensive chemical and isotopic studies (Giggenbach, 1995) of geothermal waters show that the ratios of Cl, B, Li and Cs are approximately those occurring in magmatic rocks, strongly suggesting that such waters have formed from leaching magmatic rocks, or have been in close association with such rocks for a long time.

4.1 Recent Micro-seismic and GPS Implications

Recent micro-seismic studies (Bibby et al, 1995) in the Taupo Volcanic Zone suggest that below about 7 to 9 km, the rocks under TVZ are ductile, because aftershocks are confined to above that level. This is also consistent with well temperatures in the Wairakei geothermal field (McNabb, 1992), which are at about 255°C at one kilometre depth, and increase by about 20°C in the next kilometre. If rocks become ductile at 450°C - 700°C, (Fournier, 1988, Lister, 1975) then this may occur at about 8.5 km depth.

In addition, TVZ contains about 20 geothermal fields, spaced about 15 km apart. If this spacing is due to convection cells, then the depth of these cells may be about 7.5 km in a uniform porous media. Some caution is needed, however, in performing such extrapolations, as about the upper 2.5 km of TVZ is occupied by low density pyroclastic infill (Bibby et al., 1995), characterised by seismic velocities of about 3.2 km/s. Hence, groundwater flow conditions in the upper 2.5 km may not necessarily reflect deeper flow conditions.

For the remainder of this paper we will accept the microseismic evidence, and assume that at a depth of about 7 km the rocks become ductile under TVZ, and that groundwater does not penetrate to below about 7 km. This assumption modifies the intrusive model above in two obvious ways. Firstly, the heat to chloride ratio from TVZ will be altered, as the volume of chemicals available to the groundwater has been approximately halved. Secondly, if water cannot penetrate down to the full depth H in (1) of 15 km, but only to say 7 km, then the heat production rate estimate in (1) must be reduced, since T_f is unattainable in the ductile region, and so ΔT in (1) must be reduced.

Inferences can also be drawn from geodetic measurements of extension. The heat output estimated in (1) is too great, since the average extension rate v used in (1) of 12 - 20 mm/year seems to be too great. For example, in order to obtain a heat output of about 4200 MW from (1), requires an average extension rate of about 15 mm/year (for a height H of 15 km). While local extension rates of 18 mm/year have been measured (Darby and Williams, 1991), it does seem that extension rates vary both in time and in location, but are on average somewhat less than 15 mm/year (D. Darby, personal communication). Terrestrial geodetic measurements over a 50 year time interval show an extension rate of about 8 mm/yr for the northern TVZ (Sissons, 1979), and space based GPS measurements over a one year period (1990-1991) are consistent with this result (Darby and Meertens, 1995). We shall proceed in this paper by assuming an average extension rate of 10 mm/year.

In this paper we shall assume that the intrusive model of magma passively occupying volume generated by extensional processes is basically sound. However, since we have reduced the height H in (1) from 15 km to 7 km, and v to 10 mm/year from 12 - 20 mm/year, (1) yields an energy contribution from cooling of intrusives of about 1300 MW. To account for a total output of 4200 MW then requires an additional 2900 MW. The main aim of this paper is to discuss transport mechanisms which can yield such an additional heat flow.

5. ENHANCED CONDUCTION MODEL

An enhanced rate of conductive heating is possible from the ductile region through the emplacement of magma. If the magma is intruded at temperature T_m, then it will cool down to the average temperature T at that depth in the ductile region, rather than to T_f which occurs in the brittle region. This leads to a reduction in the estimate of the total heat output from the TVZ, relative to the "intrusive" model, but will increase the conductive loss of heat from the ductile region as estimated from a linear temperature profile.

If we assume a steady temperature profile in the ductile region, then the temperature at depth z below the brittle region satisfies

$$\delta^2 \frac{\delta^2 T}{\delta z^2} = (T - T_m - \frac{L}{C}) \tag{2}$$

where

$$\delta^2 = \frac{WK}{\rho C v} = \frac{WD}{v} \tag{3}$$

and W is the width of TVZ, which we take as 30 km. In (3), D is the thermal diffusivity. For an average opening rate of 10 mm/year, δ equals 7.3 km.

Integrating (2), and imposing the boundary conditions that T equals T_f and T_m at respectively z equalling 0 and Y, yields the conductive flux J_K from the ductile region into the brittle region as

$$J_K = \frac{K}{\delta(e^{\frac{Y}{\delta}} - e^{-\frac{Y}{\delta}})}[(T_m - T_f + \frac{L}{C})(e^{\frac{Y}{\delta}} + e^{-\frac{Y}{\delta}}) - \frac{2L}{C}] \tag{4}$$

When Y / δ is small, then the conductive flux is linear, with length scale Y. However, when Y / δ is large, then the heat flow predicted by (4) becomes independent of Y, and the relevant length scale is δ, ($\sqrt{W D / v}$), or about 7.3 km when v equals 10 mm/year. Interestingly, this is close to the thickness of the ductile layer under TVZ.

Taking Y, the depth of the ductile region, as 8 km, yields J_K equalling 0.47 W/m^2 for an average opening velocity of 10 mm/year. Over the TVZ area of 5000 km^2, this enhanced conductivity yields 2300 MW. Adding this enhanced conductive output to the output from the cooling intrusions in the brittle region of 1300 MW yields a TVZ energy output of abut 3600 MW. Consequently, an additional energy source of about 600 MW is still needed, and this must be supplied by convective transport though the ductile region.

6. WATER TRANSPORT MODEL

One mechanism which can supply water and chemicals from the ductile region is through the upward buoyant rise of such a fluid through the ductile region. We are seeking a missing 600 MW of energy to be transported by this process. We shall assume that this flux is associated with the volumetric extension occurring in the ductile region.

One mechanism which may operate in the ductile region is that extension may produce volume. If this volume is initially occupied by water, either from outgassing within the ductile layer, or is supplied from beneath the ductile layer, then the rate of flow of water through the ductile layer is just the rate of volume creation within the ductile layer. This volume must be replaced eventually by intruding rock substance or magma to preserve isostacy.

If the ductile region is 8 km deep (between 7 and 15 km depth), 160 km long, and opens at an average rate of 10 mm/year, then volume is created at 0.43 m^3/s. We assume that the average temperature in the ductile region is 800°C, and the average pressure is 1600 bars (600 bars of hydrostatic and 1000 bars of lithostatic pressure). Extrapolating densities and enthalpies in the Steam Tables suggests the water substance density is about 410 kg/m^3, and its enthalpy about 3.45 MJ/kg. Thus 600 MW are available from this process. The almost exact agreement with this energy source and the missing energy must be taken as somewhat coincidental, but these calculations do suggest that buoyant transport of water through the ductile region may be important.

Within the brittle region, volume is created at 0.37 m^3/s. This corresponds to about 960 kg/s of magmatic intrusions. Consequently, the apparent magmatic water content of the intruded rocks is about 19% (176/960). To this may be added an insitu 2% water content in the intruded magma. In either situation, the apparent water content of the intruded rocks is about 20%.

In an independent chemical analysis of magmatic contents of TVZ geothermal fluids, Giggenbach (Table 3, 1995) has suggested that a separate phase of water exists in intruded magmas in TVZ. He calculates that ``andesitic" waters have a 13% water content, and rhyolitic waters a 4% water content. Unfortunately, the rhyolitic water data of Giggenbach contains considerable scatter, and so there is uncertainty about the rhyolitic water contents.

The presence of two independent predictions of such an apparent water phase within intruded magmatic rocks is very interesting, and may be relevant to mineral deposition. In Giggenbach's model, the water is assumed to occur as a foam within the rock. In this paper, the water is assumed to occur as a connected phase within the ductile region.

7. DISCUSSION AND CONCLUSIONS

This paper has briefly reviewed current work in New Zealand on the mathematical modelling of deep geothermal reservoirs in TVZ. The key new result has been a modified model of magmatic intrusions into a region of extension, which allows for a lower ductile region, in which we assume groundwater cannot travel. Since the intrusion model assumes inflowing magma replaces the volume created in the region of extension, magma must enter both above and within the ductile region. This produces an imbalance between the energy entering the ductile region, and the rate at which such energy can be transported away by conduction.

The resulting high temperature gradients, produced from a temperature gradient which varies exponentially with depth, results in very high conductive fluxes of energy. This enhanced conduction model produced an energy flux of about 0.47 W/m^2. The total energy flux from TVZ is about 0.8 W/m^2, and so conduction would play a much higher role in this model, than in any of the previous models of geothermal areas.

Added to this conductive flux is the energy flux from the cooling of intrusives in the region above the ductile region, of about 0.26 W/m^2. These two energy fluxes yield about 90% of the observed total. Given the uncertainties in the parameters used, such as opening rates, thermal conductivities, and thicknesses of the brittle and ductile regions, a 90% agreement may be close enough to support the intrusion model. If a slightly greater energy flux is needed, then this could be obtained by slightly altering some of the parameters used above, such as increasing the opening rate, or the thermal conductivity. However, if an additional energy flux is needed, in addition to that from intrusives and conduction, then we suggested the most likely source is from episodic water convection through the ductile region, perhaps from water released from the subducting Pacific Plate to the east of TVZ.

An additional apparent water phase in magmatic rocks has been deduced by Giggenbach (1995) from chemical and isotopic analysis of geothermal waters in TVZ. We suggested that such a phase may be mobile, initially occupying the extending ductile region, and eventually being replaced by intruding magma. This produces an additional energy flux of 0.12 W/m^2 to the groundwater. The total energy flux from these three mechanisms is therefore 0.85 W/m^2, which is extremely close to the observed value of about 0.84 W/m^2. Given the uncertainties in the appropriate choice of model parameters, such a close agreement was assumed to be coincidental.

A new length scale associated with the ductile region was deduced. The vertical length scale, δ in (3), is associated with enhanced conduction in the ductile region. Its depth is about 8 km, which, interestingly, is close to the thickness of the ductile region, as inferred from micro-seismic and seismic studies.

Finally, we emphasise the tentative nature of the models proposed above. While the model of groundwater circulating through the top 2.5 km of TVZ, transporting heat and chemicals, is widely accepted, the nature of transport processes below that level are at present speculative. Numerical modelling of deeper flows under TVZ has only just begun (Kissling, 1996).

REFERENCES

Anderko, A. and Pitzer, K.S., 1993. Equation of state representation of phase equilibria and volumetric properties of the system NaCl-H_2O above 573K, Geochimica et Cosmochimica Acta, 57, 1657 - 1680.

Bibby, H.M., Caldwell, T.G., Davey, F.J. and Webb, T.H., 1995. Geophysical evidence on the structure of the Taupo Volcanic Zone and its hydrothermal circulation, J. Volcanology and geothermal research, 68, 29 - 58.

Cole, J.W., 1990. Structural control and origin of volcanism in the Taupo volcanic zone, New Zealand, Bull. Volcanol., 52, 445 - 459.

Darby, D.J. and Merteens, C.M., 1995. Terrestrial and GPS measurements of deformation across the Taupo back arc and Hikurangi forearc regions in New Zealand, J. Geophys. Res., 100, 8221 - 8232.

Darby, D.J. and Williams, R.O., 1991. A new geodetic estimate of deformation in the Central Volcanic Region of the North Island, New Zealand, 34, 127 - 136.

Fournier, R.O., 1988. Accumulation of thin subhorizontal bodies of fluid in quasiplastic crystalline rocks at temperatures exceeding 350° - 400°, EOS, 69, 478.

Giggenbach, W.F., 1995. Variations in the chemical and isotopic composition of fluids discharged from Taupo Volcanic Zone, New Zealand, J. Volcanology and geothermal research, 68, 89 - 116.

Grindley, G.W., 1965. The geology, structure and exploitation of Wairakei geothermal field, New Zealand, N.Z. Geol. Surv. Bull., 75.

Hochstein, M.P., 1995. Crustal heat transfer in the Taupo Volcanic Zone (New Zealand): comparison with other volcanic arcs and explanatory heat source models, J. Volcanology and geothermal research, 68, 117 - 151.

Kissling, W.M., 1996, Large scale numerical models of TVZ-type geothermal fields, Proc. 18th New Zealand Geothermal Workshop, University of Auckland, 18, 275 -281.

Lister, C.R.B., 1975. Qualitative theory on the deep end of geothermal systems : Second United Nations Symposium on the Development and Use of Geothermal Resources, 1, 459 - 643.

McKibbin, R. and McNabb, A., 1993. Modelling the phase boundaries and fluid properties of the system H_2O - NaCl at high temperatures and pressures, Proc. 15th New Zealand Geothermal Workshop, University of Auckland, 15, 267 - 275.

McNabb, A., 1992. The Taupo-Rotorua Hot-Plate, Proc. 14th New Zealand Geothermal Workshop, University of Auckland, 14, 111 - 114.

Phillips, S.L., Igbene, A., Fair, J.A., Ozbek, H. and Tavana, M., 1983. A Technical Databook for Geothermal Energy Utilisation, LBL-12810, Lawrence Berkeley Laboratory, Berkeley, CA.

Simmons, S.F. and Weaver, S.D., 1995. Taupo Volcanic Zone, New Zealand, Special Issue, J. Volcanology and geothermal research, 68, 1 - 3, 1 238.

Stern, A.T., 1987. Asymmetric back-arc spreading, heat flux and structure associated with the Central Volcanic Region of New Zealand, Earth and Planetary Letters, 85, 265 - 276.

White, S.P. and Kissling, W.M., 1996, Including chloride and CO_2 chemistry in large scale reservoir models. Proc. 18th New Zealand Geothermal Workshop, University of Auckland, 18, 295 -301.