NUMERICAL MODELLING OF SOME GEOTHERMAL *SOURCES* IN GRADIENT MEDIA

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ABSTRACT

The geothermal flows are important in a wide range of areas, including oil recovery, geothermal development, chemical and nuclear industries, civil engineering, energy storage and energy conversion. During the last decade, the use of numerical modelling for geothermal resource evaluation has grown significantly, and new modelling approaches have been developed. In this paper we present some results from a numerical modelling of some geothermal fields caused by local volumetric distribution of sources in the earth's crust. A direct and inverse solution to defining the occupied regions by thermal sources when the heat flow on the earth's surface is known are also treated. This inverse problem is unstable and special methods of regularization of the solution are offered.

1. INTRODUCTION

Accurate knowledge of the temperature distribution in the shallow crust is of great practical improtance for understanding many geological and drilling phenomena.

A common problem in interpretation of surface heat flow data is to determine from observations the temperature distribution at depth. This reguircs knowledge of the variation in thermal conductivity as well as determination of the distribution of heat sources. If the thermal conductivity and the distribution of the heat sources were specified, the temperature distribution in a crust in conductive equilibrium could then be determined. Because of the lack of knowledge of the conductivity and the distribution of the heat sources, determination of the thermal regime of the crust from the heat flow data is an ill-posed problem.

Classical formulations of the downward continuation are generally restricted to the cases of source-free medium with known thermal conductivity, thereby circumventing the difficulty of nonuniqueness. A priori information is introduced mainly to alleviate the problem of instability. For example, Brott et al. (1981) described a continuation technique utilizing the method of equivalent point source, and Bodvarsson (1971, 1973) presented formal solutions for the downward continuation of potential fields, using the method of Fourier transform. Mareschal et al. (1985) proposed a variation of Bodvarsson's (1973) formalism. More recent works on the downward continuation of heat flow have attempted to adress the problem of variations in sources and conductivity.

We may classify these recent approaches into two groups. One group consists of the trial-and-error and Monte Carlo type formulations in which the forward problem is solved a large number of times with the model parameters sampled according to their a priori distributions. The boundedness of the parameter space ensures the stability of the solution. Examples are the formulations of Royer and Danis (1988) and of Cermak and Bodri (1986). The second group consists of those formulations which solve the inverse problem directly. Huestis (1979. 1981) described a method for finding the extremal bounds of the subsurface temperature distribution from surface gradient measurements, and Huestis and Parker (1979) and Huestis (1980) applied the Backus and Gilbert (1968, 1970) formalism to the downward continuation. Of more practical use and becoming increasingly more popular in recent years are the methods in which a priori information is incorporated in the form of soft bounds (e.g., probability distributions) and which yield optimal solutions together with the appropriate error estimates. An example is the constrained linear least squares formulation of Stromeyer (1984). A simultaneous inversion for all model parameters was given by Beck and Shen (1989) who formulated the downward continuation as a constrained least norm problem, adopting the inverse theory of Tarantola and Valette (1982a,b). Kostyanev (1991) offers a regularization method for downward continuation of constrained geothermal fields in three-dimensional gradient media.

The purpose of this paper is to present a regularization method to determine the solution of the inversion problem of defining the occupied regions by thermal sources.

2. FORMULATION AND SOLUTION OF THE PROBLEM

2.1. The direct problem

Let's treat the thermal field caused by the local volumetric distribution V of thermal sources q(M), M=(x,y,z), in a gradient media, i.e. media when thermal conductivity K(M) changes in an arbitrary way. (Figure 1). The temperature T(M) on the earth's surface is considered to be T(z=0)=0, and on the surface (z=H), $\partial T / \partial z = 0$. Thus we come to the problem:

$$\begin{cases} \operatorname{div}(K(M)\operatorname{grad}T(M)) = -q(M), & 0 < z < H, \\ T(M)\big|_{z=0} = 0, \\ \partial T(M) / \partial z\big|_{z=H} = 0. \end{cases} \qquad (1)$$

Problem (1) is a problem about an anomalus field. Our problem is to model the field caused by sources q(M). Knowing Green's function $G(M,M_0)$ (Kostyanev, **1982;** Dmitriev and Kostyanev, 1994), the thermal field can be represented in the following way:

$$T(M) = \iiint_V q(M_0)G(M,M_0)dV_{M_0}. \qquad (2)$$

If we treat a problem with constant distribution of sources q(M)=q=const and conductivity K(M)=K=const, we shall obtain:

$$T(M) = q / 4\pi K \iiint_V G_0(M,M_0)dV_{M_0}, \qquad (3)$$

where

$$G_0(M,M_0) = \int_0^\infty J_0(\lambda\rho)(e^{-\lambda(z-z_0)} - e^{-\lambda(z+z_0)} + 2e^{-\lambda H}(\mathrm{Sh}\lambda z.\mathrm{Sh}\lambda z_0) / \mathrm{Ch}\lambda H)d\lambda. \qquad (4)$$

The measurement on the earth's surface is the vertical heat flow, i.e.

$$Q(x,y)=(K.\partial T/\partial z) \qquad \text{at} \qquad z=0.$$

According to (3) and (4) we have:

$$Q(x,y) = q / 2\pi \iiint_V L(\rho, z_0)\,dV_{M_0}, \tag{5}$$

where

$$L(\rho, z_0) = \int_0^\infty J_0(\lambda\rho)\left(e^{-\lambda z_0} + \frac{Sh\lambda z_0}{Ch\lambda H}e^{-\lambda H}\right)\lambda d\lambda = \tag{6}$$

$$= -\frac{z_0}{(\rho^2 + z^2)^{\frac{3}{2}}} + \int_0^\infty J_0(\lambda\rho)\frac{Sh\lambda z_0}{Ch\lambda H}e^{-\lambda H}\lambda d\lambda$$

and

$$P = \sqrt{(x - x_0)^2 + (y - y_0)^2}.$$

The expression of $L(\rho, z_0)$ can he simplified by the substitution

$$\frac{1}{Ch\lambda H} = \frac{2e^{-\lambda H}}{1 + e^{-2\lambda H}} = 2e^{-\lambda H}\sum_{n=0}^{\infty}(-)^n e^{-2\lambda nH} \tag{7}$$

Substituting (7) in (6) we obtain:

$$L(\rho, z_0) = -\frac{z_0}{\left(\rho^2 + z_0^2\right)^{\frac{3}{2}}} + \sum_{n=0}^{\infty}(-1)^n \int_0^\infty J_0(\lambda\rho).$$

$$.\left(e^{\lambda z_0} - e^{-\lambda z_0}\right)e^{-2\lambda(n+1)H}\lambda d\lambda \ .$$

But, knowing that

$$\int_0^\infty J_0(\lambda\rho)e^{-\lambda h}\lambda d\lambda = \frac{h}{(\rho^2 + h^2)^{\frac{3}{2}}}$$

we obtain:

$$L(\rho, z_0) = \sum_{n=0}^{\infty}(-1)^n\left(\frac{2nH + z_0}{\left(\rho^2 + (2nH + z_0)^2\right)^{\frac{3}{2}}} + \frac{2(n+1)H - z_0}{\left(\rho^2 + (2(n+1)H - z_0)^2\right)^{\frac{3}{2}}}\right) \tag{8}$$

If $H >> z_0$ and the dimension of volume V, wich is occupied by geothermal sources, is also much smaller than H. we approximate the value to be considered as

$$L(\rho, z_0) \cong \frac{z_0}{\left((x - x_0)^2 + (y - y_0)^2 + z_0^2\right)^{\frac{3}{2}}} \tag{9}$$

Substituting (9) in (5) we obtain:

$$Q(x,y) = q / 2\pi \iiint_V \frac{z_0 dx_0 dy_0 dz_0}{\left((x - x_0)^2 + (y - y_0)^2 + z_0^2\right)^{\frac{3}{2}}}. \tag{10}$$

The expression (10) enables us easily to model the heat flow on the earth's surface due to a given volume V, which is occupied by geothermal sources.

2.2. The inverse problem

Knowing the heat **flow** on the earth's surface, the inverse problem of defining the region V occupied by geothermal sources can be treated. This problem is unstable and special methods of regularization of the solution are necessary (Tikchonov and Arsenin, 1979; Kostyanev, 1991). Usually, additional information on the shape of the region V is used.

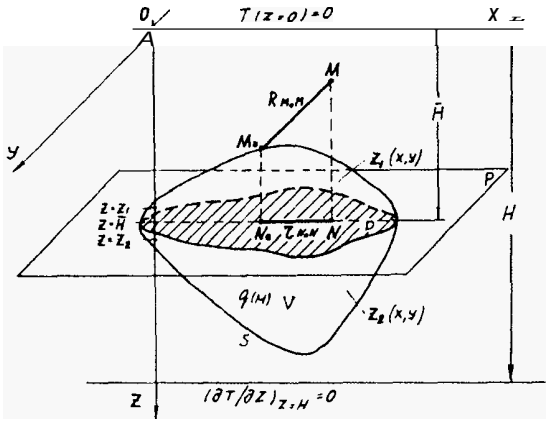

Figure 1

Let's suppose that the shape **of** V can be satisfied by the existence of a mean plane at a sertain depth H. This means that the plane $z = \bar{H}$ crosses the region V in such a way that when $z < \bar{H}$ the shape of V is described by function $z=z_1(x,y)$ and when $z > \bar{H}$ by the function $z=z_2(x,y)$ (Figure 1). Let's symbolize the section of the region V by the mean plane with D. Then formula (10) can be written in the following way:

$$Q(x,y) = \frac{q}{2\pi}\iint_D dx_0 dy_0 \int_{z_1(x_0,y_0)}^{z_2(x_0,y_0)} \frac{\partial}{\partial z_0}\frac{1}{\sqrt{(x - x_0)^2 + (y - y_0)^2 + z_0^2}}dz_0 =$$

$$= \frac{q}{2\pi}\iint_D\left(\frac{1}{\sqrt{(x - x_0)^2 + (y - y_0)^2 + z_0^2(x_0, y_0)}} - \right. \tag{11}$$

$$\left. - \frac{1}{\sqrt{(x - x_0)^2 + (y - y_0)^2 + z_1^2(x_0, y_0)}}\right)dx_0 dy_0.$$

Formula (11) completely determines the heat flow on the earth's surface, if H, $z_1(x,y)$, $z_2(x,y)$ are known. Let's we write formula (11) in operational form:

$$Q(x,y) = A\left[\bar{H}, z_1, z_2\right], \tag{12}$$

where A-nonlinear operator. The boundary of domain D is determined by the equation $z_1(x,y)=H$ (or $z_2(x,y)=\bar{H}$).

If the heat flow measurements have errors, we have an approximated $\tilde{Q}$, where $\|Q-\tilde{Q}\|\le\delta$, (δ-error of the measurement). Besides, an additional condition for the smoothness of the function must be given, for example, in the form of the minimum of a special functional:

$$\Omega\left(\bar{H}, z_1, z_2\right) = \beta\left(\bar{H} - H_0\right)^2 +$$

$$+ \sum_{i=1}^{2}\iint_D\left(P_i\left(\frac{\partial z_i}{\partial x}\right)^2 + R_i\left(\frac{\partial z_i}{\partial y}\right)^2 + S_i z_i^2(x,y)\right)dxdy, \tag{13}$$

where β is a weight coefficient, and $P_i(x,y)>0$, $R_i(x,y)>0$, $S_i(x,y)>0$ - weight functions. The value H, is a hypothetical a priori value adjacent to the depth **of** the mean plane.

Thus the inverse problem is reduced either to a problem **of** conditional extremum:

$$\begin{cases}\min_{\bar{H}, z_1, z_2}\Omega\left(\bar{H}, z_1, z_2\right), \\ \left\|A\left[\bar{H}, z_1, z_2\right] - \tilde{Q}\right\| \le \delta,\end{cases} \tag{14}$$

or to a problem of unconditional extremum:

$$\min_{\bar{H},z_1,z_2}\left[\left\|A\left(\bar{H},z_1,z_2\right)-\bar{Q}\right\|+\alpha\Omega\left(\bar{H},z_1,z_2\right)\right], \quad (15)$$

where α-cofficient of regularization, is determined by the condition

$$\left\|A\left(\bar{H}_\alpha,z_1^\alpha,z_2^\alpha\right)-\bar{Q}\right\|=\delta, \quad (16)$$

and $\bar{H}_\alpha, z_1^\alpha, z_2^\alpha$ is the solution of the problem (15) at a given α.

Thus both the shape and depth of the region V are determined. We have to note that main supposition-the existence **of** a mean plane is not applicable to all types of regions V. What shall we do if the region has not a mean plane? In this case we propose an equivalent region possessing a mean plane to be found. The same problem arises if the region is not smooth or has a complicated configuration. An equivalent region with a "smooth" shape of the surface, from the class of regions possessing a mean plane, has to be found. The described method is applied to the investigation of some geothermal sources in South West Bulgaria. These results will be further published.

3. CONCLUSIONS

The technique described consists of inverting geothermal field data by a regularization method. It is useful to obtain a numerical estimate of the temperature in the earth's crust directly from subsurface heat flow data. Numerical modelling in geothermy based on application of regularizing operators of any kind enables one to obtain an idea of the shape and the depth of the anomalous area. The more accurate the measurements and more complete a priori information, the more precise results. This method is 'simple to implement and computationally efficient.

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