Permeability Estimates at Mid Ocean Ridge Sites

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ABSTRACT

Measurements of the heat and mass flow from geothermal fields on Mid Ocean Ridges (MOR), together with estimates of the dimensions and thermal profiles of the geothermal plumes and certain assumptions concerning the isotropic and uniform nature of the porous medium, are used to obtain approximate values for the large-scale vertical permeability of the MOR system. As no drill-hole pressure and temperature data is available, an estimate is obtained for the near-surface pressure gradient imposed on each geothermal plume using a simplified model for the flow of cold seawater onto the MOR hotplate. This hotplate, at a depth h below the sea bed, produces a linear chain of such plumes of cross sectional area **A** at a distance L apart and is assumed to be a dense brine layer, just on the boil, conducting heat from a deeper magmatic heat source.

Key words: permeability, MOR, geothermal, brines, mass transfer, porous media

1. INTRODUCTION

Liquid-dominated geothermal fields behave like buoyant plumes of hot water in a porous medium subject to a pressure regime imposed on the plume by the surrounding cold water environment. For example, pressures measured below the two-phase zone in bores in the Wairakei Geothermal Field, before they had been significantly changed by discharges, revealed a hydrostatic pressure gradient at depths of about one kilometre below ground surface in the undisturbed field. The measured pressure gradient, however, was not cold hydrostatic but corresponded to that of a fluid of density $\rho^* = 872$ kg/m^3 or water at about 200 °C. Such a gradient is not unexpected, since it must also induce a cold water downflow sufficient to provide the upward fluid flux in the plumes.

If the porous medium is assumed to be uniform with constant vertical permeability k, a knowledge of this pressure gradient driving the plume and the fluid density ρ across it provides a method for estimating k from measurements of the total heat or mass output of the plume. The mass output Q and the heat output H of the plume are buoyancy driven and given by the expressions:

$$Q = \iint \frac{k}{\mu} \rho(\rho^* - \rho)\, g\, dx\, dy, \quad H = \iint \frac{k}{\mu} \rho(\rho^* - \rho)\, g\, U\, dx\, dy,$$

where μ is the viscosity of the plume fluid, and U is the internal energy of the fluid. Fischer (1964) determined H at Wairakei to be 4.5×10^8 W, and contours for temperatures at a depth of one kilometre before exploitation in the same field give an estimate of 7.6×10^{-15} m^2 for the vertical field permeable k using these formulae.

Recent measurements on the Juan de Fuca ridge reveal a series of geothermal fields with plumes roughly elliptical in shape with areas A of the order of 0.1 km^2 and a spacing L between plume centres of about 2 km. Estimates ranging from 1.7×10^9 to 1.5×10^{10} W for H and 1.2×10^3 to 8.2×10^3 kg/s for Q were obtained for the Main Vent field, but no drill-hole data is available to provide plume temperature and pressure profiles. However, substituting ρ_0, the density of cold sea water, for ρ^* will certainly give a lower bound formula for k.

In this note, we propose to substitute an estimate for the near-surface pressure gradient based on the notion that the cold seawater downflow is approximated by a three-dimensional potential flow in the region bounded at the top by the plane z = 0, and at the bottom by the plane z = h. The pressure on z = 0 is constant and at the bottom is taken to be cold hydrostatic so that no fluid flows through it. The pressure deviation from cold hydrostatic is assumed to be that generating a series of point sinks of strength 2Q spaced L apart along the ridge on z = h. Limited seismic work and some preconceived ideas about the ridge and plume structure suggest a value of 2 km for h though it is probable that the hot plate is not planar and the fluid circulation goes much deeper off the ridge axis. The enormous heat flux, and the apparent longevity of the vent field (perhaps of the order of 100 years) seem to mandate such deep circulation.

1.1 Plume pressure environment

The broad concept of the source of the MOR geothermal activity is one of a molten basalt heat source overlain by a thin crust conducting heat into a permeable medium. It is supposed that seawater is heated by conduction through this impermeable crust and perhaps through an intermediate layer of dense brine (McNabb & Fenner, 1985; McNabb, 1992) which shields the seawater from temperatures greater than about 450 °C where thermodynamic constraints and brine phase properties would cause it to precipitate most of its salt content. Seawater heated by this brine layer is protected from magmatic temperatures and is able to generate a buoyant plume convecting out heat and the same mass of salt that the cold seawater brings into it.

We assume the cold water flow can be approximated by flow into a sink of strength 2Q under each plume, where Q is the mass flow from the geothermal plumes. The flow we are seeking is given by an array of sinks in 3-space of strength $(-1)^n 2Q$ at the doubly-infinite number of points $[mL, 0, (2n+1)h]$, where m, n are integers. Such an array is associated with a pressure ϕ imposed on cold hydrostatic pressures; this is given by the expression:

$$\phi = -\frac{Q\mu_c}{2\pi\rho_0 k} \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} \frac{(-1)^n}{[(x - mL)^2 + y^2 + (z - (2n+1)h)^2]^{1/2}} \tag{1.1}$$

and at the origin this gives rise to a pressure gradient

$$\frac{\partial p}{\partial z} = \rho_0 g\left[1 - \frac{Q\mu_c}{2\pi\rho_0^2 g k h^2} f(\alpha)\right] = \rho^* g \tag{1.2}$$

where

$$f(\alpha) = \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} \frac{(-1)^n (2n+1)}{[\alpha^2 m^2 + (2n+1)^2]^{3/2}}, \tag{1.3}$$

Here, $\alpha = L/h$, μ_c is the viscosity and ρ_0 is the density of cold seawater.

2. Vertical Permeability Estimate

Let **X** denote the dimensionless parameter $\dfrac{Q\mu_c}{\rho_0^2 g k (2\pi h^2)}$.

Suppose the plume has a cross sectional area A near the sea bottom and the rising fluid has a mean density ρ and viscosity μ. The fluid mass output Q per second is then given approximately by the expression

$$Q = \frac{k}{\mu}\rho\,(\rho^* - \rho)\,g\,A = \frac{k}{\mu}\rho\,[\rho_0 - \rho_0 X f(\alpha) - \rho]\,g\,A \qquad (2.1)$$

so that

$$X\left[\left(\frac{2\pi h^2}{A}\right)\left(\frac{\mu}{\mu_c}\right) + \frac{\rho}{\rho_0} f(\alpha)\right] = \frac{\rho}{P_0}\left[1 - \frac{P}{P_0}\right] \qquad (2.2)$$

If the fluid is at a mean temperature of 350 °C near the top of the plume and Q, A, h and L are known, then this formula gives an estimate for k in terms of fluid properties Q, h, A, L and the function $f(\alpha)$. A graph of this function is given in Figure 1.

Conclusions

In the expression (2.2), the product of the dimensionless ratios $\left(\frac{2\pi h^2}{A}\right)\left(\frac{\mu}{\mu_c}\right)$ is about 12, while $\frac{\rho}{P_0} f(\alpha)$ is of the order of 6, so that the value of X is close to 10^{-2}. The measured values for Q then lead to estimates for **k** ranging from 4×10^{-13} to 3×10^{-12} m^2.

Taking the pressure as cold hydrostatic (i.e. $f(\alpha) = 0$) gives a lower bound for k whereas the point sink method probably gives an upper bound since one is modifying the sink distribution over the hotplate by moving all the sinks to one point and increasing the gradient anomaly at the origin. The argument works in cases where it is legitimate to neglect sinks distributed along the plume axis as in the case of very large Rayleigh numbers and large viscosity ratios between the hot and cold fluids. In such cases one can assume the plume flow has only a secondary interaction with the cold flow onto the hot plate.

The upper 500 m of the axis region appears to be composed of pillow basalts and sheet flows and has much lower seismic velocities than the region below, and so it could be argued that the permeability in the upper 500 m is likely to be much higher than in the lower regions. The effect of such a high permeability near the surface would be to focus the flow and decrease **A** at the top of the plumes.

The axial valley on the Endeavour segment is about 1 – 2 km wide and bounded by fault scarps with throws of 75 – 175 m, while the sea floor within the valley is cut by numerous ridge parallel fissures. Such features could lead one to believe permeabilities in the rift valley are anisotropic and higher than outside. **A** rescaling of the coordinate axes could accommodate and model anisotropy, while more complex models could be considered to deal with variable permeability. However, experience tends to make one cautious of the notion that one can "see" permeability.

NOMENCLATURE

A area of horizontal cross section of geothermal plume [m^2]
g gravitational constant [m s^{-2}]
h depth from sea bed to brine hot plate [m]
H heat flux per field [W m^{-2}]
k intrinsic permeability [m^2]
L spacing between plumes along MOR [m]
P pressure [bars absolute: 1 bar = 10^5 Pa]
Q water mass flux per field [kg s^{-1}]
t time [s]
U specific internal energy [J kg^{-1}]
x horizontal coordinate along MOR [m]
y horizontal coordinate normal to MOR [m]
z vertical coordinate, origin on sea bed [m]

Greek

μ viscosity [kg m^{-1} s^{-1}]
ρ density [kg m^{-3}]

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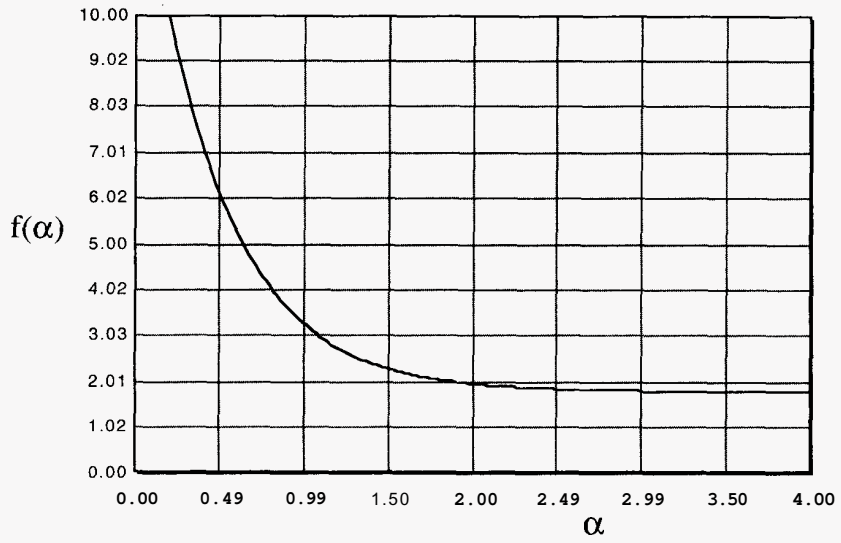


Figure 1. Graph of $f(\alpha)$ against α [see Equation (1.3)].