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AN ATTEMPT AT MODELLING HYDROTHERMAL ERUPTIONS

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ABSTRACT

Hydrothermal eruptions, like geysers, are intriguing phenomena; they are spectacular while in motion and generally unwarning while not. Their non-smooth nature makes for difficulty in both conceptual and mathematical modelling. In this paper, an attempt is made at a brief summary of some of the contemporary wisdom about hydrothermal eruptions under the notional headings of How? Why? When? and Where? This is followed by a brief flurry of equations which might be used to model an eruption in progress, together with some solutions which indicate what properties the current model possesses. The main aim of the author is to speculate, then gather feedback from those many scientists who have parts of the jigsaw in their bags of knowledge, so that some solid advance may be made in the modelling of hydrothermal eruptions.

INTRODUCTION

Occurrence

Hydrothermal eruptions have occurred in geothermal fields all around the world, and are typical features of many fields in New Zealand Recent eruption deposits are evident on the surface in most cases, while older events are recorded in extinct fields and also by subsurface formations encountered by drillholes in currently-exploited fields.

Hydrothermal eruptions are featured or are mentioned in many publications describing surface geology and surface manifestations in geothermal systems. Most of that material is descriptive in nature and sets out the evidence for eruptions; some papers record eyewitness descriptions of the few such events to take place recently when people were nearby when the eruptions started. It is not the purpose of this work to reiterate these descriptions or to provide a complete survey of eruption occurrences; the reader is referred to, amongst others, work by Bixley & Browne (1988). Browne (1986), Allis (1986), Collar (1985) and Scott & Cody (1982). These describe hydrothermal eruptions in the Taupo Volcanic Zone, but some contain references to events in other parts of the world.

From a geological perspective, historical eruptions are interesting enough, but, as considered by Bixley & Browne (1988) and Browne (1986). also have present-day and future interest in their being possible hazards in geothermal fields which are under exploitation.

Browne (1986) and others distinguish two types of eruption: natural and induced. Natural eruptions are, by definition, those which are prehistoric or have occurred more recently in unexploited fields. The prehistoric eruptions have been recorded only by identification of ejected deposits, with older craters having been filled in; in New Zealand, there is evidence of prehistoric eruptions in Kawerau, Whakarewarewa (Rotorua), Orakeikorako, Waiotapu, Rotokawa, Tikitere and Atiamuri. Vent sizes range from 5 m to 500 m in diameter and up to 500 m in depth. More recent eruptions in New Zealand fields include several at Waimangu 30-40 years after the creation of the field by the 1886 Tarawera eruption, and one other in 1973. This last eruption occurred with little precursive activity, mainly in the form of reduced spring flows in the near vicinity.

Induced hydrothermal eruptions are those which are deduced to be due to exploitation of a field and follow changes in local water level and/or near-surface reservoir fluid pressure. In New Zealand these have occurred in the Tauhara Field in 1974 and 1981 (the "Pony Club" eruptions), several near the southern boundary of the Wairakei Field in the "Craters of the Moon" area, and also at Whakarewarewa in Rotorua. Induced eruptions last up to maximum of only several hours, with many lasting only 15 minutes or so; they remove material from shallow depths, with ejected material travelling maybe only 100 m from the vent, although some may be laterally directed (e.g. at Tauhara in 1981). During all eruptions, much ejecta falls back into the vent and is in turn re-ejected; this causes considerable mixing of the source rock.

Causes

In brief summary, hydrothermal eruptions are supposed to be caused by one of 3 physical situations: the first is via the formation of a steam cap due to a drop in water level with increased steam flow to the surface; the second is due to hydraulic fracturing and brecciation allowing non-condensible gases to decrease boiling pressures near the surface; the third is due to a reduction in lithostatic pressure caused by lifting of overburden material. In the last two situations the water flashes and the subsequent large changes in specific volume causes a violent ejection of fluid and broken formation material upwards.

It should be noted that phreato-magmatic eruptions, caused by explosive flashing of groundwater by heat transfer from magmatic intrusions, are not considered here.

Prediction of eruptions appears impossible with present knowledge; there seem to be few warning precursors (or those that could be recognised as such). Changes to the point of instability in the system may be gradual; some random event (for example, seismic: see Bixley & Browne, 1988) may provide the final trigger that sets off an eruption.

The purpose of this work is to attempt to provide some qualitative and quantitative models of the conditions which might prevail just before a hydrothermal eruption, a description of events which might trigger an eruption, and a model of an eruption in progress. Much of the work is of a preliminary nature, and is presented in order to promote discussion with a view to "getting the ideas right" before refining the models.

STABILITY, INSTABILITY AND MECHANISMS

Normally, the ground and fluid in it are in a stable mode. This means that formation stresses and pore (fluid) pressures are such that small perturbations to water levels or pressures and overburden weight do not overcome the direct stresses which are keeping the formation material and fluid in place.

Analysis in terms of effective stress

The net direct force on the rock matrix material can be expressed in terms of the effective stress or pressure. Since the only escape path for any material that might be ejected is more or less upwards, this can be thought of as a pressure acting vertically upwards. The areally averaged stress, P_{eff}, is equal to the net resultant of the pressure due to the weight of the material above the point concerned, $P_{overburden}$, and the fluid pore pressure, P_{fluid}:

$$P_{eff} = P_{overburden} - P_{fluid}$$

$$= P_{atm} + \int_{z}^{0} (1-\phi)\bar{\rho}_r g dz + \int_{z}^{z_w} \phi \bar{\rho}_w g dz - P_{fluid}$$

where the pressures are measured as absolute values and z is the vertical coordinate measured positive upwards with the 'ground surface at $z = 0$ and the water level at $z = z_w$ (see Figure 1). The values $\bar{\rho}_r$ and $\bar{\rho}_w$ are the areally-averaged densities of the formation

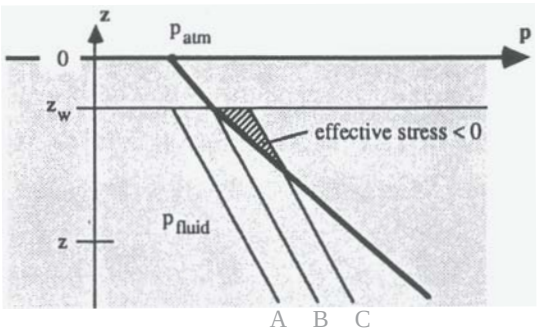

Figure 1. Schematic of system with water level at depth z_w. The overburden pressure is marked with a heavy line, while the lines marked A, B, C represent different fluid pressure profiles. The shaded area indicates where the effective stress is negative.

(rock) and water respectively. All such densities and pressures are supposed to be functions of the vertical coordinate z. It should be noted that p_{fluid} can be greater than hydrostatic-for-depth in the case where there is some form of caprock. Corrections to this formula may also be made where the fluid is partly steam and/or gas.

Where p_{eff} has a positive value, the rock grains remain compressed together (e.g. corresponding to profiles A and B for p_{fluid} in Figure 1). If at any depth, p_{eff} becomes less than zero and/or overcomes any tensile stresses which provide cohesive strength, the rock becomes effectively "buoyant" and the rock grains tend to be forced apart by the excess fluid pressure and "float", with the weight of the rock and fluid above supported by the pore fluid pressure (e.g. in the shaded region above profile C in Figure 1). That this is an unstable situation is reinforced by any sudden fluid movement upwards providing hydrodynamic lift to the matrix particles. Once some of the overburden material is removed by ejection, the effective pressure drops further and the unstable region grows in size. The fluid is now also being removed by the upward flow, so that pore pressures begin to drop; this starts to lessen the destabilisation. The adjustment of pore pressures combined with the initial removal of overburden and subsequent redeposition of falling ejecta in the vent eventually increase the local effective stress. Once p_{eff} becomes positive again, the eruption ceases. Local groundwater movement slowly adjusts pressures to a new post-eruption equilibrium, and the waiting begins again.

Note from Figure 1 that a lowering of the water surface level is a stabilising movement, reducing the size of any unstable zone, if present, where the effective stress is negative.

A more general analysis

From a physical point of view, there are several mechanisms which must occur concurrently to produce the observed or deduced phenomena associated with an eruption. A force must be generated to expel fluid and rock particles upwards from the ground. The only "stored forces" available are rock stresses and fluid pressure. For these to act directly, the previously stabilising weight of overburden would have to be removed instantaneously. The rock stresses would, however, be relieved only slowly and would contribute little to the very fast action required. The fluid pressure can only be converted to motion by significant changes in volume; the usual assumption is that water that is either liquid near boiling point, or is two-phase with a very high liquid saturation, begins to flash as pressure is reduced. The escape path is upward. The pressure propagation path is necessarily downward, since the greatest pressure changes initially possible will be near the surface where the fluid is able to escape to atmospheric conditions.

As the fluid depressurises, flashing and increasing its specific volume, large fluid velocities upwards provide hydrodynamic lift forces to the ground material above. Above the point where the lift exceeds the net gravitational and cohesive forces, the fluid/rock panicle mixture is ejected upwards; the fluid, with a much larger steam fraction as it rises up beyond the surface, continues to provide a lift force which accelerates the rock vertically. The energy is then dissipated by pressure reduction and consequent cooling effects, gravitational slowing and, probably to a small degree, friction. The speed of the motion provides litttle time for heat transfer between rock and fluid, which means that the larger rock particles will land with temperatures that are close to their in-ground values (deduced at the 1973 Waimangu eruption; see Browne, 1986). Smaller fragments, with very fine particles flowing with the water as mud, will be accelerated faster than the larger pieces and will probably be flung further; eruption breccias will tend to be better sorted the further they are from the vent (also see Bixley & Browne, 1988 and Collar, 1985).

The process can be better described as mining by steam lifting, rather than blasting, with the process taking place from above. In particular it should be noted that an upward force is not created by flashing in itself. Change of phase does not increase the pressure; under adiabatic conditions (constant enthalpy) flashing is induced only by lowering of pressure as the fluid moves upwards. Bixley & Browne (1988) note that, in a search for possible eruption triggers in the form of pressure increases beneath the Huka Formation at Wairakei, none were found; the development of a steam cap did not lead to increased pressure. The presence of a greater steam fraction at shallow levels may have led to steam fissure ventings, but these were not full-scale hydrothermal eruptions as the large volume changes caused by flashing did not occur.

Bixley & Browne (1988) also address the question of why the breakouts at Wairakei wells WK26 and WK 204 did not develop into full-scale hydrothermal eruptions. In light of the comments above, however, it is clear that conditions were indeed not right for full-scale eruptions. In the case of WK26, the formation above the feedpoint in the fissure was still in place and, apart from a narrow fissure-type venting, the unloading of the overburden necessary to reduce effective stress did not take place. The large volume changes necessary to induce a continuing high-velocity fluid flow were not available for WK204; the formation was not at near-liquid conditions, nor, again, had it been unloaded from above.

Eruption depths

The matter of "deep-seated" eruptions is addressed briefly now. It is not clear that a hydrothermal eruption can be initiated at depth. As mentioned above, even if the pressure at depth suddenly exceeds the lithostatic pressure by enough to lift a large section of the overburden, any fluid connection made with the atmosphere will almost immediately reduce the overpressure at depth and stabilise the system. Removal of the overburden has to begin close to the surface, rather than one blast removing between 200 m (Kawerau) and 500 m (Waimangu *'geyser")of material as has been suggested by previous designation of these depths as "focal depths". It is likely that these depths were the deepest levels from which rock material was removed. Browne (1986) also supports the scenario of eruptions being initiated from the surface, rather than an "explosion" at depth.

Geysers

It is also suggested here that geyser-type activity is a separate phenomenon requiring periodic flows and cycling pressures in certain parts of the reservoir; it would need a different description and analysis from that applied to hydrothermal eruptions. This view removes the Waimangu "geyser" of 1901-4 from consideration here; while the initial formation may have been as a hydrothermal eruption, later activity was not.

Effect of non-condensible gases

The initial motivation for this study was to find out what effect the characteristicsof a geothermal fluid containing significant amounts of a non-condensible gas such as CO_2 or H_2S could have on a model of a hydrothermal eruption. Previous studies (McKibbin & Pruess, 1988, 1989) of vertical counterflow in a geothermal system have indicated that a non-condensible gas such as CO_2, if present, forms its highest partial pressures in the upper part of the system. The presence of gas affects the boiling pressure and further boiling of such a fluid mixture might produce larger volume changes than for pure water and provide greater lifting velocities in vents,

Shortage of time and the discovered complexity of modelling even pure water eruption systems has delayed the testing of this speculation. Nevertheless, the existence of collapse pits caused by solution of pumice by acid gases in Rotokawa, where there is also a long history of hydrothermal eruptions, is an indication that the presence of gases makes this line of investigation an appropriate one.

Browne (1986) also supports a theory that an increase in gas content in the reservoir fluid may initiate an eruption; an injection of magmatic CO_2 into the system by volcanism, with the subsequent lowering of water pressures leading to boiling, may well provide a mechanism for hydrothermal eruptions in some fields.

A mechanism of hydraulic fracturing propagating rapidly to the surface, suggested by Hedenquist & Henley (1985), was considered by Collar (1985). The idea is that gases such as CO_2 accumulate under a local seal (not a caprock) until the pressure is great'enough to overcome the effective stress and the tensile strength of the seal and a vertical breakout occurs. However Collar found that, near the surface, the principal stress direction is horizontal and fractures would propagate horizontally there.

A NUMERICAL MODEL

Rather than trying to model the onset of a hydrothermal eruption, which would require analysis of very fast transient motion, it was decided to begin by attempting to model an eruption which is already in progress. There are two regions to be considered: the flow underground, where the rock particles are still in place and where there may be flashing and fast fluid movement, and the region above ground where the ejected material is in motion.

Experimentation with different models

Models which require account taken of several phases which are moving at different speeds, as well as complicated fluid mechanics and thermodynamics, are bound to be complex. In particular, the fluid/rock interaction forces were found to be difficult to model.

Beneath the surface, fluid is flowing at high speeds through a fractured porous material; the pressure gradient, which drives the flow, is related to frictional forces by some kind of Darcy's law. Initial experimentation using the two-phase version of Darcy's law with standard relative permeability factors was unsuccessful because vapour is modelled as being almost immobile at high liquid saturations and fluid velocities are not large. It was decided to model the fluid as a homogeneous mixture of liquid and steam governed by a sidgle-phase Darcy's law. This is probably satisfactory considering that boiling is taking place at fast rates and there is not enough time for separate stable flows of each phase to become established.

The criterion for the rock particles to be moved upward is satisfied when the hydrodynamic lift generated by the accelerating fluid flow exceeds the buoyant weight of the particles and the tensile strength, if any, of the formation. The driving force for the rock particles above ground is provided by this hydrodynamic lift with a consequent slowing of the fluid. The theory of fluidisation as well as standard hydrodynamic theory was referred to, with a representative set of describing equations being gleaned from the literature. At this stage, it is felt that modifications may yet have to be made to the basic equations; these will produce quantitative changes, but the general qualitative results given below should be representative of the description of the phenomena.

The simple model - equations of motion

In this first very simple model, the motion is supposed to be in the vertical direction only. It is assumed that the eruption is in a quasi-steady state with erosion downwards into the formation at a constant speed V (= rate of volume erosion per unit area), with the disturbance to the fluid ("flashing front") moving downwards at the same speed, at a distance below the erosion surface that is to be determined from the calculations. The front is moving down into a fluid-filled formation of porosity ϕ where the fluid, assumed motionless, has liquid saturation $S_{\ell d}$ (volume fraction of the fluid that is liquid), density ρ_{fd} and enthalpy h_{fd}. Thermal conduction, the heat content of the rock and heat transfer between the rock and the fluid are neglected. The average fluid and rock velocities are V_f and V_r respectively (note that V_f is the average interstitial or particle velocity, not the Darcy velocity). Above the ground surface, the voidage is ε (volume fraction of the flow that is fluid).

Writing velocities relative to a frame of reference moving downwards at speed V, the equations of motion can be expressed as follows (see Figure 2). In a steady state, all flow quantities depend on z only. Mass, momentum and energy are conserved. Subscripts f, r, ℓ and v refer to fluid, rock, liquid and vapour respectively.

Below ground

Conservation of mass

fluid: $$m_f = \phi\rho_f(V + V_f) = \phi\rho_{fd}V \qquad (1)$$

rock: $$m_r = (1-\phi)\rho_r V \qquad (2)$$

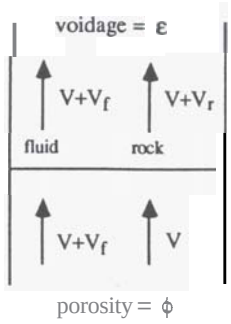

Figure 2. Velocities relative to the moving frame of reference.

where m_f and m_r are constant fluid and rock mass flow rates per unit area relative to the moving coordinate system. Here

$$\rho_f = S_\ell\rho_\ell + (1-S_\ell)\rho_v \quad \text{is the fluid mixture density}$$

and $$\rho_{fd} = S_{\ell d}\rho_{\ell d} + (1-S_{\ell d})\rho_{vd} \quad \text{from boundary conditions,}$$

where the net mass flow is $\phi\rho_{fd}V$. The rock density is ρ_r.

Conservation of momentum (Darcy's law)

The fluid flowrate per unit area, w_f, is given by

$$w_f = \frac{k}{\mu_f}\left(-\frac{dp}{dz} - \rho_f g\right) + \phi V \qquad (3)$$

where k is the vertical permeability, and the viscosity μ_f for a boiling mixture is given by the formula

$$\mu_f = \mu_\ell^{S_\ell}\,\mu_v^{1-S_\ell}$$

Mass flow rates per unit area are

$$m_\ell = S_\ell\rho_\ell w_f$$
$$m_v = (1-S_\ell)\rho_v w_f$$

with the total mass flow per unit area given by

$$m_f = m_\ell + m_v = \frac{\rho_f k}{\mu_f}\left(-\frac{dp}{dz} - \rho_f g\right) + \phi\rho_f V = \rho_f w_f$$

Combining this with the conservation of mass (1) gives

$$V_f = \left(\frac{\rho_{fd}}{\rho_f} - 1\right)V = \frac{k}{\phi\mu_f}\left(-\frac{dp}{dz} - \rho_f g\right). \qquad (4)$$

Conservation of energy

The vertical energy flux is given by

$$q_e = m_\ell h_\ell + m_v h_v$$
$$= [S_\ell\rho_\ell h_\ell + (1-S_\ell)\rho_v h_v]w_f$$
$$= \rho_f h_f w_f$$
$$= m_f h_f$$

where $$h_f = \frac{S_\ell\rho_\ell h_\ell + (1-S_\ell)\rho_v h_v}{S_\ell\rho_\ell + (1-S_\ell)\rho_v}$$

is the flowing enthalpy of the fluid. The boundary conditions at the fluid front give

$$q_e = m_f h_{fd} \qquad (5)$$

Conservation of energy implies that $h_f = h_{fd}$ i.e. the flow is isenthalpic, as expected.

The liquid saturation can be written in terms of the flowing enthalpy as

$$S_l = \frac{\rho_v(h_v - h_f)}{(\rho_l - \rho_v)h_f - (\rho_l h_l - \rho_v h_v)}$$

Solving (4) for $\frac{d}{}$ gives:

$$\frac{dp}{dz} = -\rho_f g - \frac{\mu_f \phi V}{k}\left(\frac{\rho_{fd}}{\rho_f} - 1\right) \quad (6)$$

Above ground

Notation used here is similar to that used for the flow below ground. The volume fraction occupied by the fluid is now ε, the voidage. Details of the derivation or origin of these equations is given in the Appendix.

Conservation of mass

fluid: $m_f = \varepsilon\rho_f(V + V_f) = \phi\rho_{f0}(V + V_{f0})$ (7)

where values with subscript '0' are at the eroding surface ($z = z_0$)

rock: $m_r = (1-\varepsilon)\rho_r(V + V_r) = (1-\phi)\rho_r V$ (8)

Conservation of momentum

fluid $m_f \frac{dV_f}{dz} = -\varepsilon\rho_f g - \varepsilon\frac{dp}{dz} - n\bar{f}$ (9)

rock: $m_r \frac{dV_r}{dz} = -(1-\varepsilon)\rho_r g - (1-\varepsilon)\frac{dp}{dz} + n\bar{f}$ (10)

where

$$n\bar{f} = (1-\varepsilon)\,C\,\rho_f\left[(V + V_f)\frac{dV_f}{dz} - (V + V_r)\frac{dV_r}{dz}\right] + L \quad (11)$$

Here n is the number of rock particles per unit volume of flow and the hydrodynamic lift L is given by

$$L = nF_d = \frac{3}{4}C_{ds}\frac{1-\varepsilon}{\varepsilon\phi_s d_p}\rho_f(V_f - V_r)|V_f - V_r| \quad (12)$$

where d is the diameter of a rock particle and ϕ_s is a shape function. The coefficients C and C_{ds} are defined in the Appendix.

Addition of suitable multiples of equations (9) and (10) give

$$m_f \frac{dV_f}{dz} + m_r \frac{dV_r}{dz} = -\varepsilon\rho_f g - (1-\varepsilon)\rho_r g - (1-\varepsilon)\frac{dp}{dz} \quad (13)$$

$$(1-\varepsilon)\,m_f \frac{dV_f}{dz} - \varepsilon\, m_r \frac{dV_r}{dz} = \varepsilon(1-\varepsilon)(\rho_r - \rho_f)\,g - n\bar{f} \quad (14)$$

Conservation of energy

$$m_f \frac{dh_f}{dz} = \varepsilon(V + V_f)\frac{dp}{dz} \quad (15)$$

Equation of state

All of the fluid flow below and above ground is assumed to be at saturated conditions and the equation of state is given by

$$p = p_{sat}(T) \quad \text{with} \quad (16)$$

$$\left(\frac{dT}{dp}\right)_{sat} = \frac{\left(\frac{1}{\rho_v} - \frac{1}{\rho_l}\right)(T + 273.15)}{h_v - h_l} \quad (17)$$

METHOD

Below ground, the calculation was started at the boundary where the net fluid flow is $\phi\rho_{fd}V$. The equations of motion for the flow below ground were integrated in the positive z direction (upwards), using a 4th order Runge-Kutta scheme. At each step the net hydrodynamic lift force on the rock particles [lift $= L - (1-\varepsilon)(\rho_r - \rho_f)g$ where L is given by equation (12)] was calculated. At the point where the lift is larger than some excess value determined by the cohesion of the rock material, the lift-off point was deemed to have been reached and integration then proceeded using the above-ground equations.

Severe difficulty was found in the integration of the equations for the motion above ground. A very small step-size was required to maintain convergence, and not much progress has been made so far on this part of the problem; no results are given here for the motion above ground.

Some typical results for the solution of the problem below ground are however presented with rather more confidence and are shown in Figures 3 and 4. In Figure 3 the following parameters apply:

$T_d = 150°C$, $S_{ld} = 1.0$, $\phi = 0.2$, $d_p = 0.1$ m
$k = 1.0 \times 10^{-12}$ m^2, $\rho_r = 1500$ kg/m^3, $V = 0.1$ m/s

Results are given for pressure p, temperature T, liquid saturation S_l, fluid velocity V_f and the hydrodynamic lift plotted against height z above the flashing front. The parameters are evaluated until the lift reaches the value 5000 Pa/m, which was arbitrarily chosen to represent some cohesive stress.

The pressure and temperature are related by saturation conditions. The liquid saturation decreases sharply with height while the fluid velocity increases, providing a steeply increasing lift for larger values of z. The initial decrease in the lift is caused by the decreased buoyancy offered to the rock particles as the fluid begins to flash and decrease in density, while the fluid velocities are still small.

A consequence of the calculation is an estimate of the thickness of the flashing zone under the surface; i.e. how far the fluid flashing front is moving ahead of the current eruption surface. In this case it is about 12 cm. Several other sets of calculations indicate that this thickness is proportional to the permeability, k; for example, a permeability of 10×10^{-12} m^2 or 10 darcy gives a thickness of about 1.2 m. The value of the pressure, liquid saturation and fluid velocity at the surface are unaffected by any change in permeability (all other parameters values remaining the same); this is because the state of the fluid is determined by the enthalpy, which remains constant with height and which uniquely determines saturation and fluid properties, including fluid velocity and lift.

In Figure 4, parameters are the same as in Figure 3 except for k and V, which are

$k = 10.0 \times 10^{-12}$ m^2, $V = 0.2$ m/s

As can be seen, the pattern is similar, but the flashing region is about 0.5 m thick. From the comments made above about variation of k, the thickness of the flashing zone for this value of the permeability, if $V = 0.1$ m/s, would be about 1.2 m. This and other calculations indicate that the flashing zone thickness is approximately inversely proportional to the front speed V. This could be interpreted to mean that as the eruption proceeds and some of the eruption material falls back into the vent, thereby slowing the speed of the eruption surface, the flashing front moves deeper into the system.

CONCLUSIONS

A brief review of the current thinking about hydrothermal eruptions has been given. An attempt at making a conceptual and mathematical model of an eruption in progress has been made, with mixed success so far.

There is a lot more work yet to be done on even this simple model before a satisfactory analogue is constructed for the complicated mechanism of a hydrothermal eruption. Nevertheless, a start has been made and it is hoped that it will provide a basis for discussion.

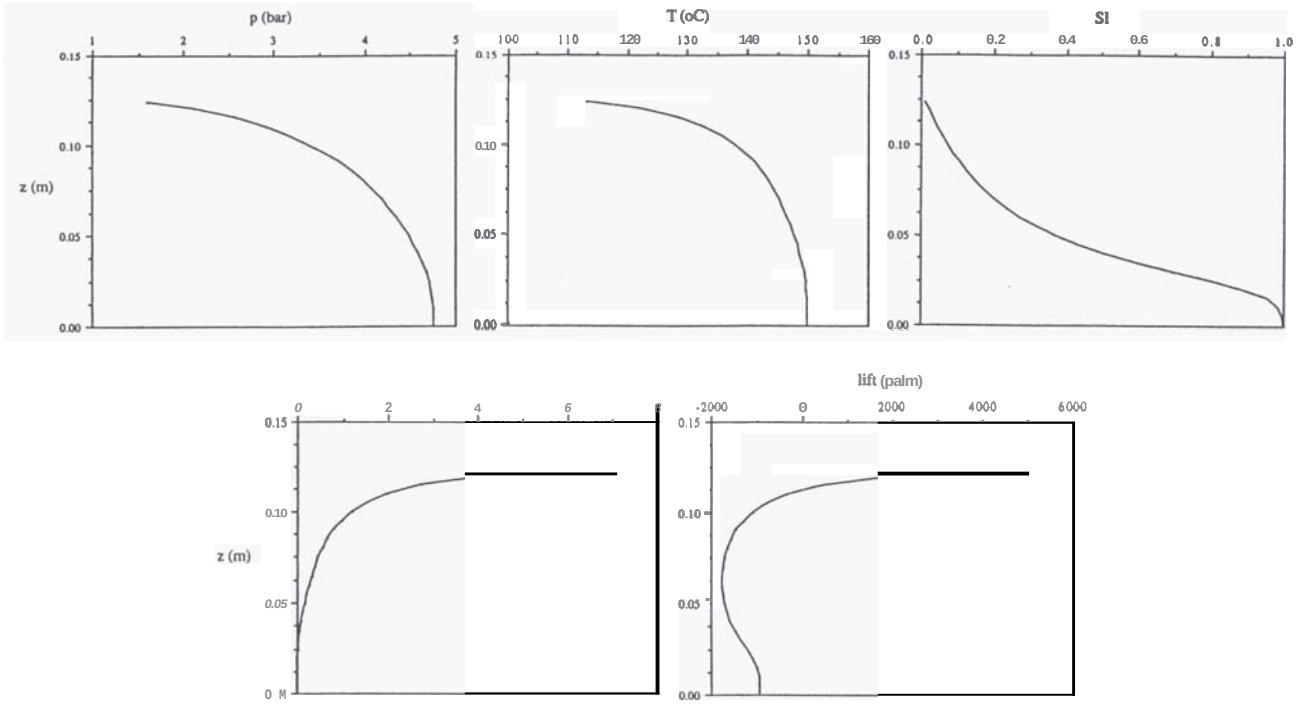

Figure 3. Values of p, T, S_g, V_f and lift plotted vs z for a typical set of *reservoir* parameters. In particular, k = 1 darcy and V = 0.1 m/s. See the text for the remaining parameter values.

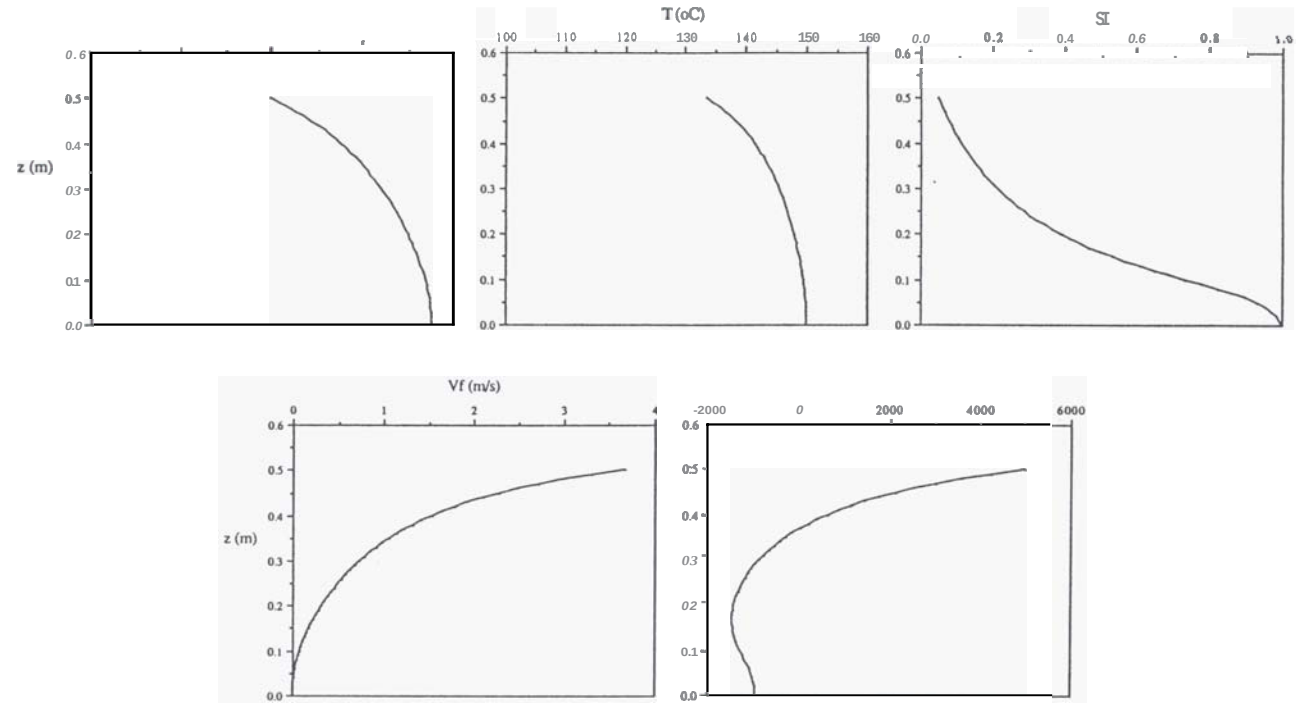

Figure 4. Values of p, T, S_g, V_f and lift plotted vs z for a typical set of *reservoir* parameters. In particular, k = 10 darcy and V = 0.2 m/s. See the text for the remaining parameter values.

APPENDIX

The basic equations of conservation of mass, momentum and energy are variously derived from first principles or adapted from those given in Davidson & Hamson (1971) and Hughes & Gaylord (1964). In the following, both tensor summation notation and vector notation are used.

Conservation of mass

fluid: $$\frac{\partial}{\partial t}(\varepsilon\rho_f) + \frac{\partial}{\partial x_i}(\varepsilon\rho_f u_i) = 0 \qquad (A.1)$$

rock: $$\frac{\partial}{\partial t}[(1-\varepsilon)\rho_r] + \frac{\partial}{\partial x_i}[(1-\varepsilon)\rho_f v_i] = 0 \qquad (A.2)$$

Conservation of momentum

fluid: $$\varepsilon\rho_f\left[\frac{\partial u_i}{\partial t} + u_j\frac{\partial u_i}{\partial x_j}\right] = \varepsilon\rho_f g_i + \frac{\partial \sigma_{ij}}{\partial x_j} - nf_i \qquad (A.3)$$

rock $$(1-\varepsilon)\rho_r\left[\frac{\partial v_i}{\partial t} + v_j\frac{\partial v_i}{\partial x_j}\right] = (1-\varepsilon)\rho_r g_i + nf_i \qquad (A.4)$$

Assume that the fluid is Newtonian and that there is a linear relationship between stress and rate of strain:

$$\sigma_{ij} = -p\delta_{ij} + \mu\left[\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} - \frac{2}{3}\delta_{ij}\frac{\partial u_k}{\partial x_k}\right] + \zeta\,\delta_{ij}\frac{\partial u_k}{\partial x_k} \qquad (A.5)$$

Here μ = shearviscosity ζ = bulk viscosity

and $$nf_i = (1-\varepsilon)\frac{\partial \sigma_{ij}}{\partial x_j} + n\tilde{f}_i \qquad (A.6)$$

where

$$n\tilde{f} = (1-\varepsilon)\,C(\varepsilon)\rho_f\left[\frac{Du}{Dt} - \frac{Dv}{Dt}\right] + n\alpha(\varepsilon)\,(u-v)\,h(|u-v|) \qquad (A.7)$$

Here g is the gravitational acceleration with components g_i, and

$$\frac{D}{Dt} \equiv \frac{\partial}{\partial t} + \mathbf{u}\cdot\nabla$$

Substitution for nf_i from (A.6) into (A.3) and (A.4) gives

fluid: $$\varepsilon\rho_f\left[\frac{\partial u_i}{\partial t} + u_j\frac{\partial u_i}{\partial x_j}\right] = \varepsilon\rho_f g_i + \varepsilon\frac{\partial \sigma_{ij}}{\partial x_j} - n\tilde{f}_i \qquad (A.8)$$

rock: $$(1-\varepsilon)\rho_r\left[\frac{\partial v_i}{\partial t} + v_j\frac{\partial v_i}{\partial x_j}\right] = (1-\varepsilon)\rho_r g_i + (1-\varepsilon)\frac{\partial \sigma_{ij}}{\partial x_j} + n\tilde{f}_i \qquad (A.9)$$

Conservation of energy

Neglecting conduction, heat content of the rock and heat transfer from rock to fluid,

$$\frac{\partial}{\partial t}\left[\varepsilon\rho_f(e_f + \tfrac{1}{2}u_j u_j)\right] + \frac{\partial}{\partial x_k}\left[\varepsilon\rho_f(e_f + \tfrac{1}{2}u_j u_j)u_k\right] = \frac{\partial}{\partial x_i}(\varepsilon u_j\sigma_{ij}) - u_j n\tilde{f}_j + u_j\varepsilon\rho_f g_i \qquad (A.10)$$

where e_f is the internal energy of the fluid.

Steady vertical flow

For steady 1-D flow in the vertical (z) direction relative to a coordinate system moving with speed V downwards,

$u_1 = u_2 = 0,\ u_3 = V + V_f,$ $\quad x_1 = x,\ x_2 = y,\ x3 = z$

$v_1 = v_2 = 0,\ v3 = V + V_r,$ $\quad \frac{\partial}{\partial t} \equiv \frac{\partial}{\partial x} \equiv \frac{\partial}{\partial y} \equiv 0,\ \frac{\partial}{\partial z} \equiv \frac{d}{dz}$

$g_1 = g_2 = 0,\ g_3 = -g$

The assumption that viscous effects are small compared with inertial effects means that the approximation $\mu = \zeta = 0$ is made. Substitution into (A.1) and (A.2) gives

$$\frac{d}{dz}\left[\varepsilon\rho_f(V + V_f)\right] = 0$$

and

$$\frac{d}{dz}\left[(1-\varepsilon)\rho_r(V + V_r)\right] = 0$$

which lead to (7) and (8).

Also $\sigma_{zz} = -p$

and $$n\tilde{f} = (1-\varepsilon)C(\varepsilon)\rho_f\left[(V + V_f)\frac{dV_f}{z} - (V + V_r)\frac{dV_r}{dz}\right] + n\alpha(\varepsilon)(V_f - V_r)h(|V_f - V_r|) \qquad (A.11)$$

Substitution in (A.8) and (A.9) lead to equations (10) and (11). The coefficient C(ε) is approximated by the value $\frac{1}{2}$ (see Davidson & Harrison, 1971).

Now to the last term in equation (A.11). The hydrodynamic lift F_{ds} exerted on a single particle of cross-sectional area A in a fluid stream of (relative) speed $V_f - V$, is equal to

$$F_{ds} = \tfrac{1}{2}C_{ds}\,A\,\rho_f\,(V_f - V_r)\,|V_f - V_r|$$

Here, $A = \pi\phi_s^2 d_p^2/4$ where ϕ_s is a shape factor (taken to be 1 for a sphere) and d_p is the diameter of a particle. The coefficient C_{ds} is a function of the Reynolds Number $Re = \frac{d_p|V_f - V_r|\rho_f}{\mu_f}$ and is approximated here by

$C_{ds} = 24\,Re^{-1} + 3.6\,Re^{-0.313}$ for $Re < 1500$

$= 0.38$ for $Re \geq 1500$

The lift exerted on a single particle in a flow where there are other particles and the voidage is ε, is

$$F_d = \frac{F_{ds}}{\varepsilon^m}$$ where in this case m is taken to be 1.

The number of particles per unit volume of the flow is n, where

$$n = \frac{1 - E}{V_p}$$

with V_p the volume of one particle, given by

$$V_p = \frac{4}{3}\pi\left(\frac{\phi_s d_p}{2}\right)^3$$

Substitution of all of these expressions into the last term of (A.11) gives

$$nF_d = \frac{3}{4}C_{ds}\frac{1-\varepsilon}{\varepsilon\phi_s d_p}\rho_f\,(V_f - V_r)\,|V_f - V_r|$$

which is equation (12).

Similar substitution into the equation for conservation of energy (A.10), and the use of equations (7) – (9) gives equation (15).

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