MODELLING THE KAWERAU GEOTHERMAL SYSTEM

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SUMMARY - This paper presents the mathematical and numerical techniques used to model the Kawerau geothermal field. Some comparisons between model predictions and measured data are presented.

1 Introduction

The Kawerau geothermal field presents some interesting challenges to modellers. There are several, poorly connected, areas of good permeability spread over large horizontal and vertical extents. The reservoir is believed to be fractured with most of the fluid flow being in the fractures.

Any model that is to be a useful aid in field management will need to be three dimensional. It will also need to treat the interaction between fluid in the fractures and the unfractured rock matrix in some manner.

There are also some features of Kawerau that make the modelling task easier. All the fluid in the field is in the liquid phase. There is no evidence of two phase regions developing under production. In fact there has been very little pressure drawdown since exploitation of the field began in 1957. This lack of pressure drawdown allows us to treat fluid flows in the field as a series of steady state problems. We assume that fluid flows change only in response to production changes and that the field responds very quickly to any change in production.

2 Derivation of Model Equations

The reservoir is viewed as consisting of rock of very low permeability split by fractures along which the fluid flows.

There are two scales on which the reservoir is modelled, a "microscopic" scale where the heat flow within a single block is considered, and a "macroscopic" scale where the reservoir as a whole is viewed. It is assumed that the average block size within the reservoir is small compared with the overall dimensions of the reservoir.

These two views are connected by demanding that the temperature at the surface of the block be equal to the fluid temperature at that location. This can be contrasted with the porous reservoir model where the temperature throughout the rock matrix is equal to the fluid temperature at the same point. The blocks are treated as a source of heat. As cold fluid flows past the blocks it does not extract all the heat from the blocks instantly, as is the case for a porous reservoir, but goes on extracting heat for some time.

A similar approach has been used previously to model thermal breakthrough in fractured reservoirs (Bodvarsson and Tsang 1982). In this work we extend the method to allow the (semi- analytic) modelling of a reservoir in three dimensions.

2.1 Assumptions

We make the following assumptions,

- all fluid is in a single liquid phase,
- pressure transients can be ignored
- instantaneous thermal equilibrium between the liquid and solids in the fracture
- infinite heat transfer coefficient between the fluid and the blocks
- no flow into or out of the blocks.

The temperature within the blocks is governed by the heat diffision equation

$$(\phi_m \rho_w c_w + (1-\phi_m)\rho_r c_r))\frac{\partial T_m}{\partial t} = k_m \nabla^2 T_m \quad (1)$$

With boundary and initial conditions

$$T_m(x,y,z,t) = T_f(x,y,z,t) \quad (2)$$

on the surface of the block

$$T_m(x,y,z,0) = T_{initial}(x,y,z) \quad (3)$$

$T_{initial}$ is the assumed initial temperature distribution of the field.

In the fractures the temperature is governed by a differential equation obtained by consideringthe heat balance in a small volume element.

$$\phi_f \rho_w c_w \frac{\partial T_f}{\partial t} = \nabla.(k_f \nabla T_f) - \nabla.(\rho_w c_w T_f V) + Q \quad (4)$$

Q is a source term obtained by integrating the heat flux across the surface of the block and dividing by the volume of the block.

$$Q = -\frac{k_m}{V_{block}} \int \int_{block\ surface} \nabla T_m d\sigma \quad (5)$$

A detailed derivation of the calculation of this source term is given in appendix A.

Equations for fluid velocity and pressure are viewed only on a macroscopic scale. At this level it is appropriate to use the equations for flow in porous media for velocity and pressure. This follows previous work (Witherspoon et al 1980) which has established that laminar flow in a fracture of width δ follows Darcys law with permeability given by $\frac{\delta^2}{12}$.

These equations are expressed as, Darcys Law

$$\vec{v} = -\frac{k}{\mu}(\nabla P - \rho_f g) \quad (6)$$

and mass conservation,

$$\frac{\partial}{\partial t}(\phi \rho_f) + \nabla.(\rho_f v) = Q. \quad (7)$$

Applying the assumption of steady state pressure and combining equations 6 and 7 gives

$$\nabla.(\frac{k}{\mu}(\nabla P - \rho_f g)\rho_f) + Q = 0 \quad (8)$$

The rate at which heat is extracted from fractured rock depends on the geometry of the rock blocks. In particular the surface area to volume ratio of the blocks is most important. For most of this preliminary work we treat the rock blocks as spheres although the case of rock slabs is also considered. Kuo et al have considered the problem of heat transfer fiom irregularly shaped blocks and shown they may be represented as spheres of equivalent thermal radius (Kuo et. al. 1976).

22 Numerical Methods

There are two distinct phases in the solution of equations 4and 8.

First a finite difference approximation to equation 8is solved to give a pressure distribution for the field. The resulting pressures are then used with equation 6 to give a velocity field. This velocity field is assumed not to vary in time, any changes in drawoff from the field necessitate the recalculation of velocities and restarting the calculation. When the calculation needs to be restarted the temperature distribution of an earlier run is used as an initial condition. It is also necessary to retain $\frac{\partial T}{\partial t}$ over a number of time steps for the calculation of the source term in equation 4.

A rectangular grid system will be used on which to specify finite difference equations. This grid is defined in a box with sides of length X_{max}, Y_{max} and Z_{max} by the equations,

$$x_i = i\Delta x, 0 \le i \le N\Delta x = \frac{X_{max}}{N} \quad (9)$$

$$y_j = j\Delta y, 0 \le j \le M\Delta y = \frac{Y_{max}}{M} \quad (10)$$

$$z_k = k\Delta z, 0 \le k \le L\Delta z = \frac{Z_{max}}{L}. \quad (11)$$

N, M and L are the number of mesh points along the x, y and z axes respectively. Solutions to the equations are obtained at discrete times $t_1, t_2, \ldots, t_n$ where $t_m = m\delta t$. Points on this grid system are represented as (x_i, y_j, z_k) and we will use the convention $T_{ijk} = T(x_i, y_j, z_k)$. Where it is necessary to differentiate between function values at different times we use T^n_{ijk} to represent the value of T_{ijk} at time step n.

Equation 8 can be expressed in finite differences as:

$$\frac{1}{\Delta x}\left(\frac{\rho_f k}{\mu}\Big|_{i+\frac{1}{2}}\left(\frac{P_{i+1jk}-P_{ijk}}{\Delta x}\right) - \frac{\rho_f k}{\mu}\Big|_{i-\frac{1}{2}} + \left(\frac{P_{ijk}-P_{i-1jk}}{\Delta x}\right)\right)$$
$$\frac{1}{\Delta y}\left(\frac{\rho_f k}{\mu}\Big|_{j+\frac{1}{2}}\left(\frac{P_{ij+1k}-P_{ijk}}{\Delta y}\right) - \frac{\rho_f k}{\mu}\Big|_{j-\frac{1}{2}} + \left(\frac{P_{ijk}-P_{ij-1k}}{\Delta y}\right)\right)$$
$$\frac{1}{\Delta z}\left(\frac{\rho_f k}{\mu}\Big|_{k+\frac{1}{2}}\left(\frac{P_{ijk}-P_{ijk+1}}{\Delta z}\right) - \frac{\rho_f k}{\mu}\Big|_{k-\frac{1}{2}} + \left(\frac{P_{ijk}-P_{ijk-1}}{\Delta z}\right)\right)$$
$$-\frac{1}{2\Delta z} g\left(\frac{\rho_f^2}{k\mu}\Big|_{k+\frac{1}{2}} - \frac{\rho_f^2}{k\mu}\Big|_{k-\frac{1}{2}}\right) = 0. \quad (12)$$

This equation is solved using Stones Strongly Implicit Method (Stone 1968).

Similarly equation 6 can be written in finite difference form

$$
\begin{aligned}
v_x|_{i+\frac{1}{2}jk} &= -\frac{k_{i+\frac{1}{2}jk}}{\mu_{i+\frac{1}{2}jk}}\left(P_{i+1jk}-P_{ijk}\right)\frac{1}{\Delta x}\\
v_y|_{ij+\frac{1}{2}k} &= -\frac{k_{ij+\frac{1}{2}k}}{\mu_{ij+\frac{1}{2}k}}\left(P_{ij+1k}-P_{ijk}\right)\frac{1}{\Delta y}\\
v_z|_{ijk+\frac{1}{2}} &= -\frac{k_{ijk+\frac{1}{2}}}{\mu_{ijk+\frac{1}{2}}}\left(\left(P_{ijk+1}-P_{ijk}\right)\frac{1}{\Delta z}-\rho_f|_{ijk}\right) \qquad (13)
\end{aligned}
$$

This definition of velocities ensures that mass is conserved with the numerical methods used.

Solution of the convection-diffusion equations for temperature proceeds as follows. By means of operator splitting, equation 4 is split into two equations

$$\phi_f\rho_w c_w\frac{\partial T_f}{\partial t} = -\nabla.(\rho_w c_w T_f V) \qquad (14)$$

a purely convective equation, and

$$\phi_f\rho_w c_w\frac{\partial T_f}{\partial t} = \nabla.(k_f\nabla T_f) + Q \qquad (15)$$

a diffusion equation.

For this work we have ignored heat conduction in the fractures and equation 15 simplifies to

$$\phi_f\rho_w c_w\frac{\partial T_f}{\partial t} = Q. \qquad (16)$$

A theoretical justification of, and error analysis for, operator splitting is given in (Strang 1968).

Given a value of T at time t, equations 14 and 16 are used to give a value for T at time $t+\delta t$ as follows. Writing 14 in terms of finite differences we obtain

$$
\begin{aligned}
T'^{n+\frac{1}{2}}_{ijk} = T^n_{ijk} &- \Delta x^{-1}[F^{n+\frac{1}{2}}_{x,i+\frac{1}{2}jk} - F^{n+\frac{1}{2}}_{x,i-\frac{1}{2}jk}]\\
&-\Delta y^{-1}[F^{n+\frac{1}{2}}_{y,ij+\frac{1}{2}k} - F^{n+\frac{1}{2}}_{y,ij-\frac{1}{2}k}] \qquad (17)\\
&-\Delta z^{-1}[F^{n+\frac{1}{2}}_{z,iik+\frac{1}{2}} - F^{n+\frac{1}{2}}_{z,iik-\frac{1}{2}}]
\end{aligned}
$$

where

$$F^{n+\frac{1}{2}}_{x,i+\frac{1}{2}jk} = \delta t\frac{1}{2}\left((\rho_w c_w v_x T^{n+\frac{1}{2}})|_{i+1jk}+(\rho_w c_w v_x T^{n+\frac{1}{2}})|_{ijk}\right)(\rho_w c_w\phi)^{-1} \qquad (18)$$

$$F_{x,i-\frac{1}{2}jk} = \delta t\frac{1}{2}\left((\rho_w c_w v_x T^{n+\frac{1}{2}})|_{i-1jk}+(\rho_w c_w v_x T^{n+\frac{1}{2}})|_{ijk}\right)(\rho_w c_w\phi)^{-1} \qquad (19)$$

$F^{n+\frac{1}{2}}_{x,i+\frac{1}{2}jk}$ is the average value of the flux in the x direction into point ijk at time steps n and $n+1$. $F^{n+\frac{1}{2}}_{x,i-\frac{1}{2}jk}$ is the average value of the flux in the x direction away from point ijk at time steps n and $n+1$.

Equation 16 can be written as

$$T'^{n+\frac{1}{2}}_{ijk} = T'^{n}_{ijk} + \delta t\frac{1}{2}(Q^n_{ijk} + Q^{n+1}_{ijk}) \qquad (20)$$

finally equation 18 is applied a second time

$$T^{n+1}_{ijk} = T'^{n+\frac{1}{2}}_{ijk} - \Delta x^{-1}[F''^{n+\frac{1}{2}}_{x,i+\frac{1}{2}jk} - F''^{n+\frac{1}{2}}_{x,i-\frac{1}{2}jk}] \qquad (21)$$

$$-\Delta y^{-1}[F''^{n+\frac{1}{2}}_{y,ij+\frac{1}{2}k} - F''^{n+\frac{1}{2}}_{y,ij-\frac{1}{2}k}] \qquad (22)$$

$$-\Delta z^{-1}[F''^{n+\frac{1}{2}}_{z,iik+\frac{1}{2}} - F^{n+\frac{1}{2}}_{z,iik-\frac{1}{2}}]. \qquad (23)$$

$F''^{n+\frac{1}{2}}_{x,ijk}$ is defined in the same way as $F^{n+\frac{1}{2}}_{ijk}$ with T_{ijk} replaced by T''_{ijk}. The algorithim used to integrate 18 has an additional step not described here. A flux correction technique due to Zalesak is applied at each half time step. By applying flux correction we avoid the artificial diffusion that is introduced by some other methods (e.g. upstream differencing) and also avoid non-physical oscillations in the solution near steep temperature gradients. It is also necessary to use a predictor-corrector iteration with equation 18 as it requires the values of T^n and T^{n+1}. Full details for the two dimensional case are given in (Zalesak 1978). Extension to the three dimensional problem is trivial.

3 Numerical Model

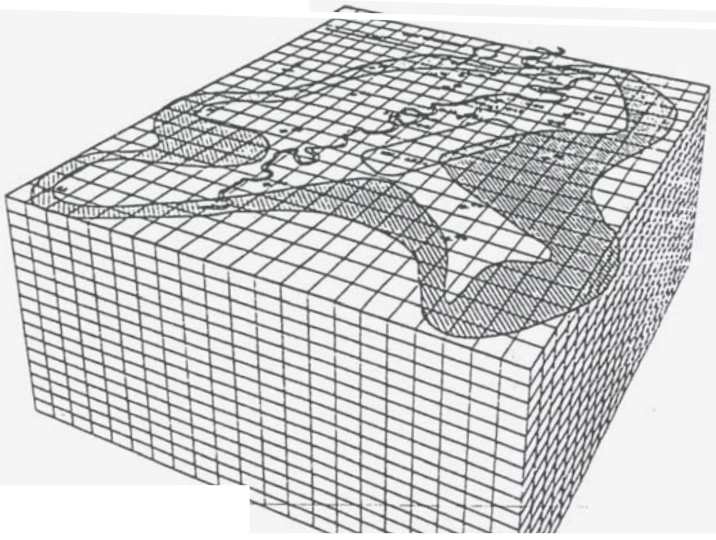

Figure 1: A Computer Model of Kawerau, one view of the 3D grid used

A three-dimensional finite difference model of Kawerau has been constructed as shown in Figure 1. The

grid used extends 4.2 km in the north-south direction, 3.7km in the east-west direction and 1.5km into the ground. The overall size of the model is almost solely motivated by the resistivity contours, although when a smaller model was initially used it was found that wells at the perimeter of the model cooled too quickly, so that cooling rates in the field have had some input. The analytical simplifications that have been made to the model allow a much greater number of grid elements than would have been possible if a full-blown two-phase geothermal simulator had been used. There are 21 divisions in the two horizontal directions, and 16 divisions depthwise, a total of 7056, giving good resolution of advancing thermal fronts.

3.1 Permeabilities

Guided by interference test results and the geology, permeabilities have been chosen for the grid elements. These are the fracture permeabilities, since the flow in blocks is not modelled here (the blocks are assumed to be of very small permeability). A primary aim of the choice of permeabilities is to reproduce the flow field inferred at Kawerau from field-wide pressure and temperature measurements. The work of Christenson (Christenson 1986) has also guided the choices of permeabilities. The permeability structure has been kept as simple as possible. No attempt has been made to assign permeabilities to computation elements on an individual basis — the small element size is to give good computational accuracy, not a finely detailed permeability structure. Hence permeabilities have been assigned to groups of elements, depending on their location. The simplicity of this approach is also motivated by the fact that the numerical problem of assigning permeabilities to match pressures is in general ill-conditioned.

For the highly permeable upper reservoir, an almost circular region with an area of approximately 1 km^2 was sketched. It includes productive shallow feeding wells and coincides with the absence of the Matahina ignimbrite unit. The bottom of this aquifer has been taken to coincide with the top of the Huka formation. A hole in the Huka formation is noted in the vicinity of KA31 and KA14.

Below the Huka formation, two aquifers are identified fiom feedpoint correlations and interference test results, one in Kawerau andesite and Te Teko ignimbrite at around 800m, and the other at 1000m and deeper in fractured basement greywacke.

3.2 Results

For the purpose of calibrating the model, exploitation of the Kawerau geothermal field was simulated between 1957 and 1984. It was not possible, without using excessive amounts of computer time, to use the exact annual production fiom each well. Rather, production from the field was approximated by 4 wells, each of which simulates the production from a number of nearby wells. The results of this modelling are summarised in Figure 2. Here we present the rate of temperature drop with discharge ($\frac{deg\,C}{Mtonne}$) verses production depth. Results are given for our model, a model presented by Grant (Grant 1977) and experimental results taken from a MWD report (MWD 1986). At first sight, the model does poorly at shallow levels when compared to temperatures taken straight from well logs. However, when this data is modified to take into account cold downflows, the fit is much better.

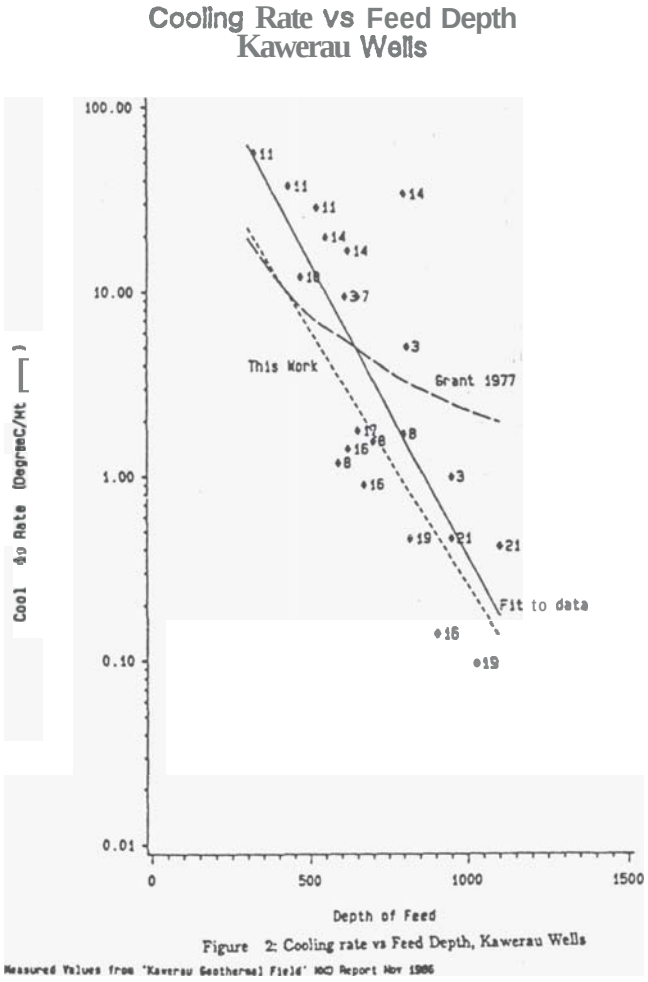

Figure 2: Cooling rate vs Feed Depth, Kawerau Wells

Measured Values from 'Kawerau Geothermal Field' MWD Report Nov 1986

4 Appendix A

In this appendix the detailed form of the source term Q used in equation 4 is derived.

The source term is defined as

$$Q = -\frac{k}{V_{block}} \int\int_{block\ surface} \nabla T_m d\sigma \qquad (24)$$

T_m is given by the solution of

$$\frac{}{at} = a^{-1} \nabla (k \nabla T_m) \qquad (25)$$

where

$$a = \phi c_w \rho_w + (1-\phi) c_r \rho_r. \quad (26)$$

This assumes the rock block is a homogeneous sphere of radius R, and has surface temperature given by the temperature of the fluid in the fractures at the block's location. This surface temperature is assumed not to change in space over the surface of the block but may change in time. Initially the block is at a constant temperature $T_{surface}(0)$ throughout.

By taking the Laplace transform of equation 25 we can solve for the temperature in transform space.

$$\bar{T} = \bar{T}_f \frac{R}{r} \frac{\sinh(\lambda r\sqrt{s})}{\sinh(\lambda R\sqrt{s})} \quad (27)$$

where

$$\lambda^2 = \frac{a}{k}. \quad (28)$$

At the surface of the block

$$\nabla_n \bar{T}_m = R^{-1}\bar{T}_f [\alpha R \coth(\alpha R) - 1] \quad (29)$$

where

$$a = \sqrt{\frac{as}{k}}. \quad (30)$$

Integrating equation 29 over the surface of a sphere radius R and dividing by the volume of the sphere gives

$$\bar{Q} = \frac{3k(1-\phi_f)}{R^2} [\alpha R \coth(\alpha R) - 1] \bar{T}_f. \quad (31)$$

It is convenient to rewrite this equation for $\bar{Q}$ as

$$\bar{Q} = \frac{3k(1-\phi_f)}{R^2} [\alpha R \coth(\alpha R) - 1] s^{-1} [s\bar{T}_f]]. \quad (32)$$

which allows $Q(t)$ to be expressed as the convolution

$$Q(t) = \frac{3k(1-\phi_f)}{R^2} \left[\int_0^t \left(\theta_3 \left(0 | \frac{i\pi(t-\tau)}{\lambda^2 R^2} \right) - 1 \right) \dot{T}_f d\tau \right] \quad (33)$$

where

$$\theta_3(0|\tau) = (-i\tau)^{-\frac{1}{2}} \sum_{n=-\inf}^{\inf} e^{\frac{-i\pi n^2}{\tau}}. \quad (34)$$

This expression for $Q(t)$ is now almost in a form suitable for numerical integration, however special consideration needs to be given to the singular behaviour of the intergrand near $\tau = t$.

The integration in equation 33 is split into two integrals, $I_1 = \int_0^{t-\delta t}$ and $I_2 = \int_{t-\delta t}^t$. I_1 has a well behaved integrand and is evaluated using the trapeziodal rule. I_2 has a singularity at t and is evaluated using the appropriate Gaussian integration rule.

Expressing Q in this form also allows a modification to equations 18-21which is very advantageous from a computational point of view. Assuming $\frac{dT}{dt}$ is constant over the time interval $(t-\delta t, t)$ I_2 is given by

$$I_2 = -\frac{3k(1-\phi_f)}{R^2} \frac{dT}{dt} \int_{t-\delta t}^t \left(\theta_3 \left(0 | \frac{i\pi(t-\tau)}{\lambda^2 R^2} \right) - 1 \right) d\tau. \quad (35)$$

The operator splitting chosen to arrive at equations 14 and 15 is not the only splitting possible, equally valid would be

$$\phi_f \rho_w c_w \frac{\partial T_f}{\partial t} = -\nabla.(\rho_w c_w T_f V) + Q_1 \quad (36)$$

and

$$\phi_f \rho_w c_w \frac{\partial T_f}{\partial t} = \nabla.(k_f \nabla T_f) + Q_2, \quad (37)$$

where $Q_1 = I_2$ and $Q_2 = Q - Q_1$.

This does not alter the form of equations 18-20 but does change equation 19 to

$$F^{n+\frac{1}{2}}_{x,i+\frac{1}{2}jk} = \delta t \frac{1}{2} (\rho_w c$$

$$F_{x,i-\frac{1}{2}jk} = \delta t \frac{1}{2} \left((\rho_w c_w v_x T^{n+\frac{1}{2}})|_{i-1jk} + (\rho_w c_w v_x T n + \frac{1}{2})|_{ijk} \right)$$

$$(\rho_w c$$

where $I\frac{dT}{dt} = Q_1$

Formulating the equations in this way allows a larger time step δt to be used with equations 18-20 without affecting the stability of the solution.

5 References

Christenson, B.W. (1986) ***Hydrology and Fluid Chemistry of the Kawerau Geothermal System,*** Chapter 5 and ***Fluid*** Inclusion ***Studies,*** Chapter 6 in ***The Kawerau Geothermal Field: Contributions from the 1982 Kawerau Seminar and Other Recent Scientific Investigations,*** DSIR Geothermal Report Number 10 (August 1986), a report compiled for the Gas and Geothermal Trading Group, Ministry of Energy, Wellington.

Grant **M.A.** (1977) *Temperature Changes and Patterns at Kawerau* Geothermal Circular Number MAG16, Applied Mathematics Division, DSIR, Wellington

Kuo, M. C. T., **P.** Kruger, and **W.** E. Brigham (August 1977) *Shape-Factor Correlations for Transient Heat Conduction from Irregular Shaped Rock Fragments to Surrounding Fluid,* Paper **77-HT-54,** Proceedings, AICHE-ASME Heat Transfer Conference, Salt Lake City, ***UTAH***

Lai C. **H.,** G. S. Bodvarsson, P. A. Witherspoon **(1985)** A *Second-Order Upwind DifferencingMethod*

For Nonisothermal Chemical Dansport In Porous Media, Internal *Report* **Earth** *Sciences Division,* **Lawerence** *Berkely Laboratory, University of* **California,** *Berkely*

Stone, **H. L. (1968)** *Iterative Solution of Implicit Approximations of Multidimensional Partial Differential Equations,* **SIAM** *J Numer.* Anal.,Vol **5, 530-558,** *September* **1968**

Strang, G. (1968) *On the Construction and Comparison of Difference Schemes,* **SIAM** *J Numer.* Anal.,Vol **5,506-517,** *September* **1968**

Witherspoon, **P.** *A., J.* **S. Y. Wang, J. E. Gale (1980)** *Validity of Cubic Law for Fluid Flow in a Deformable Rock Fracture,* **Water** *Resources Research* **16** (6), *pp* **1016-1024.**

Zalesak, S. **T. (1979)** *Fully Multidimensional Flux-Corrected Transport Algorithms for Fluids, J Comp.* **Phys. Vol31** *Num* **3 379-408,** *June* **1979**